

An Innovative Algorithm for Carrier-Phase Navigation

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BIOGRAPHY

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ABSTRACT

An algorithm is described that uses the carrier-phase measurements to propagate the position and clock states forward in time with a minimum of computational burden. The algorithm uses the change in the carrier-phase measurement over the propagation interval in a unique way. Specifically, rather than treat the change in the phase measurements as range difference measurements, they are treated as range error

measurements. This allows the same gain values that are computed in the low-rate position and clock computations to be used in the high-speed position and clock updates. The change in the phase measurements is corrected by the change in satellite position and may, if one wishes, also be corrected by the change in satellite clock and the expected change in user position and clock. The technique is highly accurate and can be used in all navigation modes, including stand-alone GPS, DGPS and RTK implementations.

NavCom has developed two variations of the technique. In one variation, the "maximum availability" mode, the change in the L1 carrier-phase measurements is used and the elevation angle threshold is lowered to a few degrees. (If available the L2 carrier-phase measurements are used as well, simply to reduce the effect of the measurement noise.) This allows the navigation to be maintained with good accuracy when the navigation might otherwise be aborted due to too few satellites at good elevation angles or due to loss of L1 or L2 carrier-phase measurements from signal blockage. Over short intervals all the slowly changing factors can be ignored in the computation. This includes effects from the ionosphere, troposphere, satellite clock, DGPS or RTK corrections. Thus, the computation is very simple and efficient.

In the second variation of the technique, the "maximum accuracy" mode, all of the above effects are included in an attempt to make the position propagation as accurate as possible. The intent is to allow the technique to be used for extended intervals. Such an approach allows the primary full-computation to be run at a lower rate, which reduces the computational load without sacrificing any significant accuracy.

INTRODUCTION

GPS navigation and positioning in all its forms relies increasingly upon the carrier-phase measurements. Almost all GPS receivers will, at a minimum, smooth the code measurements with the carrier-phase measurements,

particularly if the carrier-phase measurements are not separately incorporated into the position solution process. In the very first GPS receivers, the changes in the carrier-phase measurements were measured directly and used in the measurement processing to improve the velocity accuracy. The pseudorange code measurements were the primary measurements. Today the situation is dramatically different. To obtain the highest GPS accuracy, the carrier-phase measurements are used almost exclusively. RTK (Real Time Kinematic) differential carrier-phase GPS provides centimeter accuracy and typically uses the code measurements, if at all, only to assist in the resolution of the whole-cycle phase ambiguities.

Given the high-accuracy that GPS now provides, applications that require high-rate position information have become increasingly common. (Papers presented in this conference session describe 50 and 100 Hz raw data output rates.) The desire for high-rate position information, together with increasing demands for associated processing within the GPS receivers, has put increasing demands upon the internal processing capabilities of the receivers. Even with rapidly increasing computational capabilities within the receivers, this has put a premium on efficient algorithms for the position and velocity solution. Described below is a very efficient carrier-phase processing technique, which can be used to compute the position and velocity at high rates with minimal computational requirements.

BACKGROUND

The algorithm developed below describes the use of the change in carrier-phase measurements to propagate the receiver position and clock states forward in time at a high-rate with a minimum of computational burden. By treating the change in the phase measurements as range difference measurements, they can be processed using the same gain values that are computed in the associated low-rate process.

The algorithm described assumes the use of a least-squares processing technique at the major, i.e. low rate, epochs. This is not essential. The algorithm could be easily modified to use weighted-least-squares or even a Kalman filter approach to the position solution.

The propagation algorithm can be used in virtually any mode of GPS navigation, from the lowest accuracy stand-alone mode to the highest accuracy RTK mode of operation. The change in the carrier-phase measurements, when treated as a range measurement error, allow a direct computation of the change in receiver position (and clock). Because of the high accuracy of the phase measurements, the RTK accuracy is not compromised; and in any other mode of operation the change in position

accuracy will exceed the accuracy of the low-rate position solution.

The algorithm depends upon values which are computed as part of the low-rate major epochs to minimize the high-rate computation at the minor epochs. A simple classical least-squares algorithm is used to illustrate the computation required at the low-rate major epochs.

LOW-RATE LEAST-SQUARES PROCESSING

At the low-rate, the least squares solution is used to compute the major epoch position updates. For each satellite, the measurements equation for all modes is of the form:

$$z = \mathbf{h}\mathbf{x} + \mathbf{h} \quad (1)$$

Where: \mathbf{x} is the state correction vector (change in position and clock) value to be computed; \mathbf{h} is the measurement sensitivity vector, which characterizes the effect of any errors in the state vector upon the measurement; η is the measurement noise; and z is the measurement innovations, i.e. the difference between the measurement and the expected value of the measurement given the current estimate of the state vector (position and time).

The only difference between modes is the specific measurements used in computing the measurement innovations (also referred to as the pre-fix residuals). When differential code measurements are used, the measurements are first corrected by the differential corrections from the reference site. When the RTK mode is used, the carrier-phase measurements are used after first being corrected by the carrier-phase corrections from the reference site and then modified by the appropriate whole-cycle ambiguity value.

Equation (1), when expanded into matrix form to represent the set of equations from all tracked satellites, becomes:

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

The least-squares solution to equation (2), which minimizes the effect of the noise vector, \mathbf{n} , is given by:

$$\mathbf{x} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{z} \quad (3)$$

Where the superscript, T , represents the transpose, and the superscript, -1 , represents the inverse.

The matrix operations can be performed to give simpler forms of equation (3):

$$\mathbf{x} = \mathbf{A}\mathbf{H}^T\mathbf{z} \quad (4)$$

or

$$\mathbf{x} = \mathbf{Bz} \quad (5)$$

Where: $\mathbf{A} = (\mathbf{H}^T \mathbf{H})^{-1}$ and $\mathbf{B} = \mathbf{A} \mathbf{H}^T$.

The matrix \mathbf{B} has four rows, corresponding to the three position coordinates and the clock. It has as many columns as there are satellite measurements available. It is stored for subsequent use in the high-rate propagation computation.

It is also useful, as will become apparent, to be able to compute the post-fix residuals directly from the innovations or pre-fix residuals. Given the correction to the state vector, \mathbf{x} , the post-fix residuals, \mathbf{r} , are given by:

$$\mathbf{r} = \mathbf{z} - \mathbf{Hx} \quad (6)$$

But using equation (5) to replace the state update, \mathbf{x} , in equation (6) gives:

$$\mathbf{r} = \mathbf{z} - \mathbf{HBz} \quad (7)$$

This in turn can be simplified to:

$$\mathbf{r} = \mathbf{Sz} \quad (8)$$

Where: $\mathbf{S} = (\mathbf{I} - \mathbf{HB})$.

The matrix, \mathbf{S} , maps the pre-fix residuals into the post-fix residuals. \mathbf{S} is square and the number of rows and columns are equal to the number of satellite measurements. It is also stored for use in the high-rate propagation computation.

The high-rate computation uses the change in the carrier-phase measurements to compute the change in position (and clock) over the high-rate epoch intervals. Two modes of operation are described below: a “maximum availability” mode and a “maximum accuracy” mode.

HIGH-RATE “MAXIMUM AVAILABILITY” PROPAGATION OF POSITION

Having stored the \mathbf{B} matrix used in equation (5) above, the change in carrier phase over the high-rate epoch can be used to propagate the position forward in time with high accuracy and with minimal computations. The high accuracy is a result of the low noise in the carrier-phase measurements. The first computation required is to compute the innovations (pre-fix residuals), \mathbf{z} for use in equation (5). The change in the measured carrier-phase (delta phase) for each satellite is a major component of the innovations. Generally, only one correction to the delta-phase measurements is required to make the innovations accurate enough to maintain centimeter

accuracy over major epoch intervals of one second. Specifically, one must subtract from the delta phase the change in the radial distance to the satellite over the high-rate epoch interval. The specific equation to compute the innovations for each satellite is:

$$z_i = (\mathbf{f}_i - \mathbf{r}_i) - (\mathbf{f}_{i-1} - \mathbf{r}_{i-1}) \quad (9)$$

Where the \mathbf{f} represents the phase measurement scaled by the wavelength and the \mathbf{r} represents the range to the satellite. The subscript i represents the current epoch and $i-1$ the prior epoch.

The difference between the current phase measurement and the range to the satellite can be stored for use in the subsequent epoch. The satellite position computation should be optimized for this high-rate computation. There are a number of methods described in the literature to optimize this computation.

Most of the normal corrections to the innovations that are required at the low-rate major epochs are not required for the innovations at the high-rate. This is because they generally change by less than the centimeter level over the one-second interval between the major epochs. This generally includes: 1) the satellite clock error; (2) the ionospheric and tropospheric refraction; and 3) the differential corrections from the base station(s). Of these factors, the largest may well be the satellite clock errors. They can contribute an error which can approach one centimeter over a one-second major epoch interval. But it is quite easy to incorporate the satellite clock correction if desired. It is simply the clock frequency offset times the high-rate epoch interval.

In the “maximum availability” mode of operation, the reliability can be improved and the noise can be decreased by using measurements which might not be included in the major epoch computations. For example, the change in carrier-phase may be quite accurate even from satellites that have too low an elevation angle to be included in the major epoch computation. They may be excluded from the slow-rate major epoch because the multipath is too large. But the multipath changes slowly, and it will generally not adversely affect the change in carrier-phase over a one-second major epoch. Similarly, while the major epoch may remove ionospheric effects by using both L1 and L2 measurements to refraction correct the measurements, the high-rate epoch can reduce the noise by averaging the L1 and L2 change in carrier phase or by only using the L1 measurement when the L2 is unavailable. Of course, the \mathbf{B} matrix, computed at the low rate and stored for use at the high rate, must include all the satellites to be used in the high-rate computation.

Several options are available, in terms of adjusting the high-rate innovations for receiver motion and receiver

clock frequency. Generally, the clock rate can be ignored; and the **B** matrix row used to compute the receiver clock error can be deleted. However, it may be desirable to remove the major effect of receiver clock error to avoid numerical problems. This can be accomplished by subtracting from each innovation value the average across all innovation values. The receiver position change across the high-rate epoch interval can be estimated from the velocity and removed from the innovations if desired, but it is not required. The equation:

$$\mathbf{v} = \mathbf{x} / \Delta t \quad (10)$$

can be used in alternate ways depending on whether or not the innovations were adjusted for the receiver motion using the velocity. If the innovations were adjusted for the receiver velocity, equation (10) will yield the correction to the velocity vector. If the innovations did not include an adjustment, equation (10) will yield the entire velocity vector. In either case the velocity computed will be very noisy if the computation represented by equation (10) is done at a high rate. Thus, the velocity so computed should be smoothed or put into a position-locked loop to yield a smoothed velocity output.

One other significant problem remains to be addressed. What does one do during the high-rate computation if a new satellite arises, or if one of the satellites being tracked sets, or if measurements are lost? It is highly desirable to avoid the computational burden required to compute a revised **B** matrix.

If a new satellite arises and its measurements become available for processing at the high-rate, one can simply ignore the measurements until the next low-rate epoch. This has very little penalty and is a very simple solution.

Loss of measurements, whether due to obstruction or a setting satellite is not so simple. If the measurements drop below four (three with altitude hold), all one can do is to extrapolate the position based upon velocity. However, if four or more measurements remain, either the **B** matrix must be reconstructed or the missing measurement must be synthesized in some manner. A method of synthesizing the measurements is described, which is equivalent to recomputing the **B** matrix. However, it is a much simpler computation.

The **S** matrix, used above in equation (8) to map the innovations into post-fix residuals, has many useful properties. In a prior paper [1], a simple but effective RAIM was developed using the **S** matrix. The same matrix can be used here to synthesize one or more pre-fix residuals (innovations) for any satellites whose signal has been lost. For each satellite whose measurement has been lost, the post-fix residual is set to zero. Equation (8) can

then be used to solve for the pre-fix residual(s) that would give those null post-fix residual(s). Because the post-fix residuals are zero for the missing measurements, they clearly will have no effect upon the computed solution. For example, if only one measurement has been lost, the specific post-fix residual for that satellite is set to zero and the row in equation (8) associated with that satellite can be used to solve for the required innovation value. Specifically, for satellite *i* whose measurement has been lost:

$$r_i = 0 = \sum_j s_{ij} z_j \quad (11)$$

Where the subscript *i* designates the row of the **S** matrix and the subscript *j* designates the specific elements of that row. Equation (11) can be solved for the innovation value, z_i associated with the missing measurement.

$$z_i = - \frac{\sum_{i \neq j} s_{ij} z_j}{s_{ii}} \quad (12)$$

When two measurements are missing, two equations of the form of equation (11) result and there are two unknown innovations that need to be determined. Similarly, three missing measurements would result in three equations with three unknowns. If more than three measurements are missing (and there are still at least four available), it may be more efficient to recompute the **B** matrix or to simply extrapolate on the velocity.

“MAXIMUM ACCURACY” PROPAGATION OF POSITION

The “maximum accuracy” mode is similar to the “maximum availability” mode except that an attempt is made to correct for even the minor effects which cumulate slowly with time. This mode is used to provide centimeter accuracies over time intervals from 10 to 30 seconds. For example, it can be used as an intermediate mode between a 10 second low-rate computation and a high-rate maximum availability mode at a 1/10th second interval. Thus, it could provide significant computation relief from the normal one second process.

In order to correct for ionospheric effects, the only satellites included are those where both L1 and L2 carrier-phase measurements are available. To avoid excessive cumulation of multipath and tropospheric effects, the minimum elevation angle of the satellite is set to at least 10 degrees.

Like the maximum availability mode, the innovations are corrected for the satellite motion; and the receiver motion may or may not be estimated. If it is estimated, equation

(10) is used to correct the velocity, or else it is used to estimate the entire velocity. Unlike the maximum availability mode, the innovations are corrected for all the known small effects. These include: (1) the ionospheric effects which are corrected using both the L1 and L2 measurements; (2) the satellite clock frequency which is available from the satellite message; (3) the change in any measurement corrections being supplied by a reference station or a network of reference stations, including any implied geometry change in those corrections; (4) the change in any tropospheric refraction being modeled at either the reference (if not already included in the received correction value) or at the user receiver.

Ignoring the satellite clock frequency offset which contributes to a change in phase over the time interval can create as much as one centimeter error each second. The change in the corrections from the reference site or site networks can also contribute a small error. Generally, these changes are small but they can cumulate over several seconds into a significant error. Usually, they are easy to correct, but subtle effects can sometimes arise depending on the mode of operation. For example, corrections from NavCom's StarFire Global DGPS network are given in Cartesian coordinates of the satellite. These corrections need to be mapped into line of site corrections to the range before forming the difference to avoid losing the effect of the changing geometry.

Van Grass and Lee [2] described similar high-accuracy carrier-phase algorithms. However, they transmit the measurement data from the reference site, which increases the computational load on the user receiver and complicates the correction for the changing geometry.

SAMPLE RESULTS FOR A DROPPED SATELLITE

The use of equation 12 is compared to the recomputation of the **B** matrix for a specific numerical example. The particular data set used to make the comparison is the same set that was used to generate sample results in the RAIM paper [1] presented at last year's conference.

No.	Innovations (z)	Innovations - Bias
1	-218.547079	-0.005753025
2	-218.5458158	-0.004489820
3	-218.5325198	0.008806160
4	-218.5530649	-0.011738897
5	-218.5352066	0.006119423
6	-218.5275850	0.013740939
7	-218.5351474	0.006178554
8	-218.5475043	-0.006178300
9	-218.5405446	0.000781382
10	-218.5487924	-0.007466415

Table 1: Innovations for 10 satellites

In Table 1 the innovations for 10 satellites are listed as they were computed for a specific 0.1 second interval in a stationary receiver. The last column has subtracted off the average as a first estimate of the clock offset. This improves the numerical resolution of the solution. This column of innovations is multiplied by the **B** matrix to obtain the state update. As given in the prior paper, the **B** matrix and state updates are given in Table 2 and Table 3 respectively. The frequency row (before transpose) has been deleted since the frequency offset of the receiver is not generally used. (The value is retained in Table 3.)

-0.143565506	0.221103528	-0.292642275
-0.147379499	0.31698798	0.127526791
0.158511159	-0.127927715	-0.518258574
0.169278373	0.175486107	0.361331686
-0.342541775	-0.07931904	0.667336355
0.30093104	-0.032684788	0.240712086
-0.001723545	-0.202855377	0.04796674
0.206546896	-0.202765339	-0.090291622
-0.213114646	-0.089507034	0.101673489
0.012983064	0.021404563	-0.645251705

Table 2: B Matrix (Transpose) sans the Frequency Row

Delta North	0.001385075
Delta East	-0.007046667
Delta Up	0.005448203
Delta Clock	0.001629056

Table 3: Solution to the Innovations in Table 1

This answer can be compared to the result obtained when one satellite's measurement is assumed to be lost. Satellites 7 and 8 have innovations of about the same magnitude but of opposite sign and are about midway between the largest and smallest in magnitude. If we assume the measurement from the 7th satellite is lost, we can compare the solution obtained using the **S** matrix via equation (10) with the original solution and with the solution obtained by recomputing the **B** matrix for the remaining nine satellites.

The **S** matrix as given in the prior paper is given below in Table 4. Equation (10) is used to compute an innovation value for the 7th satellite using the 7th row of this matrix. This requires the computation of the dot product of the row with the innovations in Table 1 (excluding the seventh element) and dividing that dot product by the negative of the element of **S** in the 7th row and 7th column. Performing the computation gives new innovation values of -218.55404 and 0.004730925 which can be compared to the original values for the seventh row of Table 1.

0.61558	-0.3378	-0.09293	-0.05848	0.00559	0.10086	0.05103	0.09207	-0.09162	-0.28423
-0.3378	0.54399	0.08014	-0.27027	-0.10215	-0.02931	0.10684	0.14591	-0.04596	-0.09131
-0.09293	0.08014	0.65739	0.05339	0.14801	-0.08285	-0.15667	-0.23683	-0.06311	-0.30651
-0.05848	-0.27027	0.05339	0.51916	-0.01134	-0.38257	0.03675	-0.07212	0.09184	0.09373
0.00559	-0.10215	0.14801	-0.01134	0.35009	0.03069	-0.22069	-0.00788	-0.34819	0.15573
0.10086	-0.02931	-0.08285	-0.38257	0.03069	0.54924	-0.0936	-0.24797	0.07816	0.0774
0.05103	0.10684	-0.15667	0.03675	-0.22069	-0.0936	0.75165	-0.21805	-0.19441	-0.06291
0.09207	0.14591	-0.23683	-0.07212	-0.00788	-0.24797	-0.21805	0.69548	-0.06334	-0.08727
-0.09162	-0.04596	-0.06311	0.09184	-0.34819	0.07816	-0.19441	-0.06334	0.72421	-0.08766
-0.28423	-0.09131	-0.30651	0.09373	0.15573	0.0774	-0.06291	-0.08727	-0.08766	0.59308

Table 4: S Matrix for the 10 satellites

Clearly, the two computed values are close to, but slightly different, from the original innovation values computed directly from the 7th satellite's measurements.

To compute the revised solution, the value in the seventh element of the last column of Table 1 is replaced with the new value of **0.004730925**; and this column is multiplied by the original **B** matrix given in Table 2 above. This gives the new state updates as:

Delta North	0.001387577
Delta East	-0.00675301
Delta Up	0.005378765

Table 5: Solution with measurement 7 missing

The change in the solution resulting from the drop from 10 satellites to 9 is about 0.3 millimeters.

This solution is now verified by using the **H** matrix as given in the prior paper and computing a new **B** matrix based on the original measurements with the seventh satellite deleted. This revised **B** matrix without the frequency row is given in transposed form in Table 6.

With the exception of the missing row (column before transposition), the **B** matrix has only changed slightly from the original values give in Table 2 above.

-0.1434711	0.2348836	-0.2958577
-0.1471293	0.3458552	0.1207247
0.1581188	-0.170231	-0.5082022
0.1693780	0.1854361	0.3590323
-0.3429518	-0.138841	0.6811758
0.3007299	-0.057931	0.2467237
0.2060528	-0.261620	-0.0763811
-0.2135198	-0.1419610	0.1139474
0.0127925	0.004409	-0.641163

Table 6: Revised B matrix (transposed)

To get the direct solution the innovations in Table 1, with the seventh value deleted, is multiplied by the **B** matrix in Table 6. The result is:

Delta North	0.001388319
Delta East	-0.006753015
Delta Up	0.005376841

Table 7: Direct solution with measurement 7 missing

The difference between the solution obtained by synthesizing the 7th satellite's measurement (Table 5) and that obtained by the full rigorous computation (Table 7) is only one micron in height and is due to the limitations in the numerical precision.

CONCLUSIONS

A simple and very efficient algorithm has been described for propagating the position forward in time at a high rate. Because the algorithm uses the change in carrier-phase measurements at the high-rate epoch interval, it is very accurate and does not limit the accuracy when used in even the highest precision RTK mode. Two variations were described. One mode, the "maximum availability" mode, is designed to make maximum use of any available satellites. It is optimized to propagate the position forward for only short intervals of time, typically only one second. A second mode of operation, the "maximum accuracy" mode, is designed to propagate the position forward for multiple seconds. It attempts to correct for all known factors which affect the accuracy of the propagation.

New satellites which arise during the high-rate position propagation can be ignored until the next low-rate epoch without any significant impact on the navigation. However, a satellite whose measurements have been lost must be deleted from the position computation. A new algorithm, which makes use of the **S** matrix to synthesize the measurement innovation has been described and illustrated.

REFERENCES

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