

# A New Three-Frequency, Geometry-Free, Technique for Ambiguity Resolution

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## BIOGRAPHY

Ronald R. Hatch was one of the founders of NavCom Technology, a John Deere Company, and is currently the Director of Navigation Systems. Prior to joining NavCom, he worked as a GPS consultant with a number of companies and government agencies. Included among these were Leica, Honeywell, Northrop, NASA and the FAA. He worked for 3 years at Johns Hopkins Applied Physics Laboratory, five years at Boeing and 23 years at Magnavox. Ron's recent work has concentrated on high-accuracy applications of GPS. He is the author of many survey and navigation papers and has over a dozen patents involving GPS processing techniques. Ron received a B.S. degree in Math and Physics from Seattle Pacific College. He has served in a number of positions with the Institute of Navigation (ION) including Chair of the Satellite Division and in 2001-2002 as President. Ron was the 1994 recipient of the Satellite Division Kepler Award and in 2000 received the Thomas L. Thurlow award from the ION. He was elected a Fellow of the ION in 2000.

## ABSTRACT

A new technique for ambiguity resolution at long distances is described. It uses the code and carrier measurements on three frequencies in an unusual way. Specifically, it uses an averaging method to arrive at an accurate ambiguity-resolved and refraction-corrected measurement that largely overcomes the disadvantage of the close spacing between the L2 and the L5 frequencies. Of course, it works better when the second and third frequencies are farther apart, such as the Galileo L5 and E6 frequencies. The technique is unique in that there is no requirement to resolve the ambiguities of the fundamental L1, L2 and L5 carrier phase measurements. Instead a wide-lane, but noisy, refraction-corrected carrier phase measurement is formed from two of the three wide-lane carrier phase differences formed from the difference of pairs of the fundamental phase measurements. These differences are ambiguity resolved using ionospheric-matching code measurements and are then combined into a refraction-corrected composite measurement. While this wide-lane composite is quite noisy, it can be

smoothed with a refraction-corrected, composite measurement with much lower noise.

The ambiguities of this low-noise composite measurement are not required since it is simply used to smooth the noise in the wide-lane refraction-corrected composite. By not requiring the stepping from one ambiguity-resolved carrier phase measurement to another, it is largely immune to clock differences at the different frequencies which can sabotage the stepped approach. In addition, because the initial ambiguity resolution is done with wide-lane combinations, the reliability of the ambiguity resolution is robust and relatively insensitive to the presence of small code-carrier biases.

The geometry-free approach of individually resolving the ambiguities removes the tropospheric refraction from the ambiguity resolution problem. Thus, the final smoothed, refraction-corrected composite measurement is insensitive to both ionospheric and tropospheric refraction effects. Though the smoothing process may require some minutes to reach the optimal accuracy level, the result should significantly extend the ranges over which RTK results can be obtained without requiring the modeling of the ionosphere.

## INTRODUCTION

The smoothing of the code measurements with the carrier phase measurements is routinely done in most GPS or GNSS receivers. In single-frequency receivers this smoothing is normally limited to time constants of one to two minutes to avoid biasing the smoothed measurements with a component of the diverging ionospheric effects.

When two or more frequencies are available, the code measurements can be smoothed with a linear combination of carrier phase measurements which match the ionospheric refraction effects. This allows significantly longer averaging time constants. Of particular interest, the wide-lane carrier phase measurements formed by differencing the individual carrier phase measurements can be used to smooth a frequency weighted composite code measurement which matches the ionospheric refraction effects of the wide-lane carrier phase

measurement. After sufficient smoothing, the resultant smoothed code measurement can be used to set the ambiguities of the carrier phase difference measurement, assuming, of course, that the code measurement biases are small.

When three or more frequencies are available, two independent carrier phase difference measurements can be formed and the ambiguities resolved via the matched smoothing process just described. But having two ambiguity-resolved carrier phase difference measurements allows us to obtain a refraction corrected result with the appropriate linear combination. Unfortunately, when two of the carrier frequencies are close together, the linear combination that removes the ionospheric effects greatly amplifies the phase noise present in the measurements. This can be overcome by smoothing the composite, refraction-corrected, ambiguity-resolved, carrier phase measurement with another refraction-corrected carrier phase measurement which is constructed to minimize the noise without regard to the ambiguity resolution problem [1]. However, some additional accuracy may be achieved by a final residual ambiguity resolution.

The details of this process with examples for GPS and for Galileo are described below.

## GENERAL CONSIDERATIONS

There are a few general issues which need to be addressed before the process described in the introductory section is detailed. First, for simplicity, the equations will be written as if there is no differencing of measurements across sites. In fact, theoretically, the process can be used on single-site measurements. However, there can be significant code versus carrier biases in the transmission from the individual satellites which would prevent the process from working on a site by site basis. But, given measurements at known sites around the world, it may be possible to measure any code or carrier biases and characterize them as a function of the angle to the receiver site relative to the satellite fixed coordinates. Such a calibration process would allow single site processing. Without calibration, the equations can be applied directly to the measurements either differenced across sites or the measurements of a given site after adjustment with corrections generated at a reference site.

Second, the equations are written as if there is no differencing of measurements across satellites. The receiver front-end filters may create, in effect, a different clock reference at the different received frequencies. This can create a bias between the wide-lane phase measurements (the difference of the reference clocks at the two frequencies) and the matching frequency-weighted code measurements (a weighted average of the

clocks at the two frequencies). If this bias is large it could lead to incorrect ambiguity resolution. This problem can be avoided by subtracting the measurements from a given satellite or from an average across all satellites.

Since the troposphere affects the measurements by the same amount at each frequency, the ambiguity resolution, and refraction correction processes are transparent to tropospheric effects. Specifically, the ambiguity resolution process and the forming of the refraction-corrected linear combination will leave the tropospheric component of the measurements unchanged. The advantage of the “geometry-free” approach is that the tropospheric induced range errors do not adversely affect the ambiguity resolution process.

## RESOLVING THE WIDE LANE AMBIGUITIES

The first step in the process of obtaining a low-noise refraction-corrected and ambiguity-resolved carrier phase measurement is to form at least two wide-lane carrier phase differences and to resolve the ambiguities in those wide-lane measurements. To avoid the repetition of the same equations applied to a number of different frequencies, the process is described first using the code and carrier phase measurements at three general frequencies labeled  $f_a$ ,  $f_b$  and  $f_c$ .

The code measurements,  $P_a$  and  $P_b$ , at the first two of these frequencies are given by:

$$P_a = \rho + I / f_a^2 \quad (1)$$

$$P_b = \rho + I / f_b^2 \quad (2)$$

In these two equations,  $\rho$  is the geometric range (including tropospheric refraction induced error) and  $I$  is the ionospheric range error as a function of the inverse frequency squared.

In similar fashion we can write the scaled carrier phase measurements,  $\Phi_a$  and  $\Phi_b$ , as a function of the raw phase measurements,  $\phi_a$  and  $\phi_b$  as:

$$\Phi_a = (\phi_a + N_a)c / f_a = \rho - I / f_a^2 \quad (3)$$

$$\Phi_b = (\phi_b + N_b)c / f_b = \rho - I / f_b^2 \quad (4)$$

In these two equations  $N$  represents the unknown cycle ambiguity at the frequency indicated by the subscript and  $c$  is the speed of light.

By computing the frequency-weighted average of equations (1) and (2) we can reduce the multipath corruption of the code measurements somewhat and get a

new dependence on the ionospheric refraction error. Specifically:

$$\overline{P}_{ab} = \frac{f_a P_a + f_b P_b}{f_a + f_b} = \rho + \frac{I}{f_a f_b} \quad (5)$$

This frequency-weighted code measurement matches the ionospheric error of the wide-lane carrier phase difference measurement.

$$(\phi_a - \phi_b)\lambda_{a-b} + (N_a - N_b)\lambda_{a-b} = \rho + \frac{I}{f_a f_b} \quad (6)$$

The wavelength designated,  $\lambda_{a-b}$ , is the wavelength of the difference frequency.

Differencing equation (6) from equation (5) and dividing by the difference wavelength gives a direct measure of the wide-lane ambiguity.

$$N_{a-b} = N_a - N_b = \frac{\overline{P}_{ab}}{\lambda_{a-b}} - (\phi_a - \phi_b) \quad (7)$$

Since the wide-lane ambiguity does not change as long as phase-lock is maintained by the receiver tracking loops, this value can be averaged over time (smoothed) to get an increasingly accurate ambiguity value. The smoothing can be done with an expanding average filter.

$$N'_{a-b,n} = \frac{1}{n}(N_{a-b} - N'_{a-b,n-1}) + N'_{a-b,n-1} \quad (8)$$

(The  $n$  indicator of the amount of smoothing is dropped in subsequent equations.) After smoothing, this value can be plugged into equation (6) to give an ambiguity resolved, wide-lane carrier phase measurement.

$$\Phi_{ab} = (\phi_a - \phi_b + [N'_{a-b}\lambda_{a-b}]_{rnd}) = \rho + \frac{I}{f_a f_b} \quad (9)$$

The amount of smoothing required to ensure that the correct wide-lane ambiguity is determined in equation (8) is a function of the wavelength of the difference frequency (the longer, the better). Table 1 gives the associated wavelength of the difference frequencies for three different system choices of the three frequencies. The  $f_a$  and  $f_c$  frequencies are chosen to correspond to the L1 and L5 frequencies of GPS, which are common to the L1 and E5 frequencies of Galileo. Three different choices of the middle frequency,  $f_b$ , are considered, the GPS L2 frequency (which is 51.15 MHz above the L5 frequency), the Galileo E6 frequency (which is another 51.15 MHz

above the L2 frequency), and a third frequency another 51.15 MHz above the Galileo E6 frequency.

Mid. Freq	$\lambda_{a-b}$	$\lambda_{b-c}$	$\lambda_{a-c}$
GPS L2	0.8619	5.8610	0.7514
Galileo E6	1.0105	2.9305	0.7514
1.3299 GHz	1.2211	1.9537	0.7514

**Table 1: Difference frequency wavelengths**

### REFRACTION CORRECTED WIDE LANE PHASE MEASUREMENTS

The last term in equation (9) above represents the corruption induced in the wide-lane measurement by the ionospheric refraction. This error needs to be removed. The first step in removing this ionospheric error is to form a second ambiguity resolved wide-lane carrier phase measurement from the three available primary phase measurements.

Only two of the three possible wide-lane differences which can be formed are independent. Therefore, it makes sense to form the two wide-lane differences which have the widest lane width, since those will take the least amount of averaging time to determine the ambiguity values. Thus, assuming  $f_a > f_b > f_c$  the wide lane carrier phase represented by differencing the  $f_b$  and  $f_c$  measurements should be formed next. We do not need to go through the above derivation all over again. The appropriate equation representing the ambiguity-resolved, wide-lane carrier phase measurement from the  $f_b$  and  $f_c$  carrier phase measurements is directly analogous to equation (9) and is obtained simply by replacing the original subscripts with the appropriate new subscripts.

$$\Phi_{bc} = (\phi_b - \phi_c + N'_{b-c})\lambda_{b-c} = \rho + \frac{I}{f_b f_c} \quad (10)$$

Having determined the  $N_{a-b}$  and  $N_{b-c}$  ambiguities, the  $N_{a-c}$  ambiguity can be obtained simply by summing the first two. Thus, the third ambiguity-resolved wide-lane carrier phase difference is easily obtained.

Given two equations which show a different dependence on the ionospheric refraction, it is a relatively straightforward process to generate a linear combination of the two measurements which removes any ionospheric refraction error. Specifically, equation (10) is multiplied by  $f_c/f_a$  and then subtracted from equation (9).

$$\Phi_{RC} = \frac{f_a}{f_a - f_c}\Phi_{ab} - \frac{f_c}{f_a - f_c}\Phi_{bc} = \rho \quad (11)$$

This is the desired result. Specifically, equation (11) gives us a refraction-corrected, ambiguity-resolved, carrier phase measurement with no ionospheric refraction corruption.

## THE NOISE PROBLEM

Unfortunately, the process of forming both the wide lanes and the refraction correction cause the noise in the measurements to be substantially amplified. The noise amplification is independent of which two of the three possible wide-lane combinations are selected for the refraction correction process. In fact, assuming that it were possible to resolve the ambiguities on the individual carrier phase measurements (designate them  $\Phi_a$ ,  $\Phi_b$  and  $\Phi_c$  respectively—see equations (3) and (4)), each of the three possible pairs of ambiguity resolved, wide-lane measurements when refraction corrected—as in equation (11)—give rise to the same equivalent equation.

$$\Phi_{RC} = \frac{f_a^2}{(f_a - f_b)(f_a - f_c)} \Phi_a + \frac{f_b^2}{(f_b - f_a)(f_b - f_c)} \Phi_b + \frac{f_c^2}{(f_c - f_a)(f_c - f_b)} \Phi_c = \rho \quad (12)$$

By picking the frequencies of L1 and L5, which are common to GPS and Galileo, as the  $f_a$  and  $f_c$  frequencies respectively, we can study the noise induced as a function of the choice of the middle frequency,  $f_b$ . Two different assumptions regarding the noise will be explored. First, we will assume that there is noise equal to one centimeter in each of the phase measurements when scaled by their respective wavelengths. Then we will assume equal noise in each of the phase measurements (19 degrees). Scaling these phase errors by the wavelength results in noise of 1.0,  $f_a/f_b$ , and 1.34 centimeters respectively. These noise levels are quite pessimistic but are intended to include the effects of substantial multipath noise.

The expected noise in the refraction-corrected measurement is given by the square root of the sum of the squares of the three coefficients in equation (12), weighted by the respective noise. The noise resulting from the three previously considered choices of the middle frequency are explored. Table 2 gives the resultant noise for these three choices of the middle frequency when equal phase noise is one cm on each.

Mid. Freq.	C1	C2	C3	RSS
GPS L2	17.89	-84.71	67.82	109.98
Galileo E6	20.97	-53.88	33.91	67.03
1.3299 GHz	25.34	-46.94	22.61	57.94

**Table 2: Amplification of one cm. noise**

Clearly, the Galileo E6 signal is substantially better than the GPS L2 signal for minimizing the noise of the refraction-corrected, ambiguity-resolved, wide-lane carrier phase measurements.

When the noise is assumed to be 19 degrees of phase in each of the primary phase measurements, the advantage of the Galileo E6 frequency is even more pronounced. This is shown in Table 3 below where the weighted coefficients and RSS values are given.

Mid. Freq	wC1	wC2	wC3	RSS
GPS L2	17.89	-108.71	90.82	142.78
Galileo E6	20.97	-66.38	45.41	83.11
1.3299 GHz	25.34	-55.61	30.27	68.20

**Table 3: Amplification of 19 degrees of phase noise**

The noise in these refraction-corrected, ambiguity-resolved, wide-lane, composite measurements rivals that of the raw code measurements. However, the noise of the composite phase measurement is still generally better than a refraction corrected code measurement would yield. This is particularly true when biases and noise averaging are considered.

## CARRIER SMOOTHED CARRIER

Just as the code measurements can be (and usually are) smoothed using carrier phase measurements, the refraction-corrected, ambiguity-resolved, wide-lane carrier phase measurements generated via equation (10) can be smoothed to substantially reduce the noise. To do the smoothing we need a refraction-corrected carrier phase measurement with minimum noise, but the ambiguities do not need to be resolved.

Assuming one centimeter of phase noise in each of the primary scaled phase measurements, the minimum-noise refraction-corrected combination can be determined from three constraints. Specifically, the values of the coefficients,  $a$ ,  $b$  and  $c$ , used to multiply the primary phase measurements at the three frequencies must satisfy:

$$a + b + c = 1 \quad (13)$$

$$a / f_a^2 + b / f_b^2 + c / f_c^2 = 0 \quad (14)$$

$$a^2 + b^2 + c^2 = \min \quad (15)$$

The first of these three equations ensures the range measurement is not scaled. The second ensures that the ionospheric refraction error is canceled, and the last ensures that minimum noise amplification occurs. (If non-equal noise is present in the three phase measurements, this last equation needs to be modified appropriately.)

Solving the first equation for  $c$  gives:

$$c = 1 - a - b \quad (16)$$

Plugging this value for  $c$  into equation (14) and solving for  $b$  gives:

$$b = \frac{f_a^2}{f_a^2 - f_c^2} - a \frac{f_a^2(f_b^2 - f_c^2)}{f_b^2(f_a^2 - f_c^2)} \quad (17)$$

Using obvious definitions for these frequency functions, this can be simplified to:

$$b = F_a - aF_b \quad (18)$$

Inserting the values of  $c$  and  $b$ , given in equations (16) and (18) respectively, into equation (15) gives the value  $v$  which we wish to minimize. After simplification we get:

$$v = (1 - 2F_a + 2F_a^2) - 2a(1 - F_a - F_b + 2F_aF_b) + 2a^2(1 - F_b + F_b^2) \quad (19)$$

Taking the derivative with respect to  $a$ , setting it to zero, and then solving for the value of  $a$  gives:

$$a = \frac{(1 - F_a - F_b + 2F_aF_b)}{2(1 - F_b + F_b^2)} \quad (20)$$

The values of  $b$  and  $c$  can now be obtained by back substitution into equations (18) and (16). The values of  $a$ ,  $b$ , and  $c$ , obtained when the middle frequency assumes the three different values used earlier, is given in Table 4. The final column gives the estimated noise amplification of the equal noise primary measurements. This shows that, if the primary ambiguities could be resolved, the GPS L2 frequency actually results in a slightly lower refraction corrected noise than the Galileo E6 frequency.

Mid. Freq.	a	b	c	RSS
GPS L2	2.3269	-0.3596	-0.9673	2.546
Galileo E6	2.2691	-0.0245	-1.2446	2.588
1.3299 GHz	2.1065	0.3135	-1.4200	2.560

**Table 4: Coefficients for refraction correction with minimum noise**

Defining the minimum-noise refraction-corrected value as  $\Phi_M$ , its value is computed as:

$$\Phi_M = a\Phi_a + b\Phi_b + c\Phi_c \quad (21)$$

Where  $\Phi_a$ ,  $\Phi_b$ , and  $\Phi_c$  are the respective carrier phase measurements scaled by their wavelength and with the whole-cycle ambiguities estimated (i.e. the ambiguities may not be correct).

Both the value computed from equation (11) and the value computed from equation (21) contain a measurement of the refraction corrected range. Thus, when differenced, the value yielded will be a function of the multipath noise on the three frequencies and a constant bias error caused by any incorrect ambiguities used in equation (21). Thus,

$$O = \Phi_{RC} - \Phi_M \quad (22)$$

If this offset difference,  $O$ , is smoothed in an increasing-average filter, its value will approach the negative value of the bias error present in equation (21). Specifically, the smoothed offset is given by:

$$S_n = \frac{1}{n}(O - S_{n-1}) + S_{n-1} \quad (23)$$

where the value of  $n$  increases by one at each measurement epoch.

This smoothed bias value can then be added back onto the value from equation (21) to give an increasingly accurate refraction-corrected carrier-phase measurement without any bias. Specifically, a smoothed refraction-corrected measurement,  $\Phi_s$ , is obtained from:

$$\Phi_s = \Phi_M + S_n \quad (24)$$

If the noise in the phase measurements were random (white), the expected noise in equation (24) would decrease from the noise of equation (11) or (12) (as given in the last column of Table 2) toward the noise of equation (21) (as given in the last column of Table 4) in a one over the square root of  $n$  trajectory. However, the noise is dominated by multipath effects, which are not white. The noise averaging depends upon the autocorrelation function of the multipath and receiver measurement noise. The initial positive autocorrelation causes the noise averaging to be slower than independent noise; however, after several minutes, the autocorrelation goes negative which causes the noise to average out faster than independent noise. It should also be noted that, unlike code multipath effects, the carrier phase multipath has an equal distribution of positive and negative error

and should average to zero over time. The net result is that after 15 to 30 minutes of averaging one could expect the residual noise to approach a few centimeters.

### OPTIONAL RESOLUTION OF THE AVERAGE AMBIGUITY

As indicated previously, only two of the wide-lane ambiguity-resolved carrier phase measurements are independent. Thus, the third, wide-lane ambiguity value can be computed from the first two. This leaves one degree of freedom in the whole-cycle ambiguities of the primary carrier phase measurements. Specifically, given the wide-lane ambiguity values, if any one of the whole-cycle ambiguities present in the primary carrier phase measurements can be determined, then the other primary ambiguities can be computed as well. Assume, for example, the value of  $N_a$  used in equation (3) and subsequently in equation (21) has been estimated to be one whole cycle larger than its true value. Since the value of the wide lane whole cycle ( $N_a-N_b$ ) has been determined correctly in equation (7) above, the value assigned to  $N_b$  will also be one cycle too large. In similar fashion, the value assigned to  $N_c$  will be one cycle too large. Thus, constrained by the correct wide-lane ambiguity values, any estimation error in one of the primary ambiguity values will cause an equal error in each of the other primary ambiguity values, i.e. the average value of the three primary ambiguity values will be off by one. This characteristic allows us to compute the effect of an improper estimate of a whole-cycle ambiguity on the refraction corrected value obtained from equation (21) above. In Table 5 below, the refraction corrected wavelength, i.e. the range error in equation (21) as a result of a one-cycle estimation error in the whole cycle ambiguities, is given. The contribution to this range error from each frequency is also given using the coefficients of equation (21) found in Table 4.

Mid. freq.	$a\lambda_a$	$b\lambda_b$	$c\lambda_c$	$\lambda_{RC}$
GPS L2	0.4428	-.0878	-.2465	0.1085
Galileo E6	0.4318	-.0058	-.3172	0.1089
1.3299 GHz	0.4008	0.0707	-.3619	0.1097

**Table 5: Refraction corrected wavelength**

In each case, the effect of an error of one cycle in the choice of the primary ambiguities results in a refraction-corrected error introduced into equation (21) of almost 11 centimeters. Since the refraction correction in equation (11) or (12) is independent of errors in the primary ambiguities and dependent only on the wide-lane ambiguities, it is clear that the difference between the two, represented by the offset in equations (22) and (23), will have errors which will cancel out the error in equation (21). But this means that after sufficient smoothing the smoothed offset value of equation (23) will

reach a steady state value which, when divided by the refraction-corrected wavelength (final column of Table 5) and rounded to the nearest integer, will give the number of whole cycles by which to correct the primary whole cycle ambiguity values. That is:

$$N_a = N_a + [S_n / \lambda_{RC}]_{rnd} \quad (25)$$

$$N_b = N_b + [S_n / \lambda_{RC}]_{rnd} \quad (26)$$

$$N_c = N_c + [S_n / \lambda_{RC}]_{rnd} \quad (27)$$

Thus, after sufficient smoothing time, estimated at 15 to 30 minutes, it should be possible to correct the whole-cycle ambiguity values used in equation (21) and step to the final optimal accuracy representative of that equation as indicated in the final column of Table 4. An additional patent has been applied for which covers the process needed to resolve the final refraction corrected ambiguity.

### ALTERNATE METHODS FOR COMPUTING THE NOISE OFFSET VALUE

There are several alternate means of generating the offset value of equation (22) above which needs to be smoothed to reduce the noise and to quantize it to the nearest whole cycle. It turns out that the three primary measurements can be combined to eliminate the ionospheric refraction effect, leaving only the range (and noise). Or the three measurements can be combined to eliminate the range leaving the ionospheric effect (and noise). That leaves only one degree of freedom to solve for another variable. That independent combination can eliminate both the range and the ionospheric refraction effect and leave only a specific combination of the noise in the three measurements. The particular combination of noise which results is that which is uncorrelated with either the range or the ionospheric effect. This means that any measurement combination which cancels out the effect of both range and ionosphere will result in a scaled version of this same noise combination. (This noise offset will be biased by any errors in the ambiguity values.) Since the offset value defined in equation (22) is one such measurement combination that cancels out both range and ionospheric effects, it is a particular instance of such a scaled noise combination.

There are, however, a number of other methods (all equivalent) which can be used to generate the offset value. One could use different combinations of measurements to solve for the ionospheric refraction effect and then difference two of these to generate a scaled offset value. Or one could use different combinations of measurements to generate different refraction corrected measurements of range and then difference any two of these to generate a scaled offset value. Finally, one could in one step generate a

combination of measurements which eliminates both the range and ionospheric effects (as is done below).

Andrew Simsky, in Reference 2, describes a method of generating a composite measurement from three frequencies which has no range dependence and no ionospheric refraction effects. Specifically, it has only noise and multipath dependence. It is thus another instance of a scaled version of the offset value given in equation (22). His equation (adapted to my notation) is:

$$O_s = \lambda_c^2(\Phi_a - \Phi_b) + \lambda_b^2(\Phi_c - \Phi_a) + \lambda_a^2(\Phi_b - \Phi_c) \quad (28)$$

I have labeled this as  $O_s$  since it turns out it is simply a scaled version of the offset difference given in equation (22) above, which is successively smoothed in equation (23). Scaling this equation a bit differently can yield coefficient values on a par with the coefficients in equation (12) above. Specifically, form the value:

$$O_s = f_a f_b f_c \left[ \left( \frac{\Phi_a - \Phi_b}{f_c^2} \right) + \left( \frac{\Phi_c - \Phi_a}{f_b^2} \right) + \left( \frac{\Phi_b - \Phi_c}{f_a^2} \right) \right] \quad (29)$$

Clearly, the range component is canceled in each pair of primary measurements. The ionospheric components are cancelled between terms and the residual is due entirely to noise and multipath plus the effect of any whole-cycle ambiguity estimation errors. Simplifying the equation gives:

$$O_s = \frac{f_a(f_b^2 - f_c^2)}{f_b f_c} \Phi_a - \frac{f_b(f_a^2 - f_c^2)}{f_a f_c} \Phi_b + \frac{f_c(f_a^2 - f_b^2)}{f_a f_b} \Phi_c \quad (30)$$

Table 6 gives the coefficients of equation (30) in numerical form when  $L1=f_a$ ,  $L5=f_c$  and the middle frequency,  $f_b$ , is that shown in the first column. For subsequent use, the last column shows the bias in the value which would result from a positive one cycle error in the three primary carrier phase whole cycle ambiguities—labeled  $\lambda_{amb}$  in the last column in Table 5. This is simply a measure of the value of the scale factor which is dependent on the particular way in which the range and ionospheric effects were eliminated. The range is cancelled since the coefficients sum to zero.

Mid. Freq.	$C_a$	$C_b$	$C_c$	$\lambda_{amb}$
GPS L2	13.112	-71.085	57.973	-.09142
Galileo E6	25.711	-74.047	48.336	-.14972
1.3299 GHz	37.856	-77.009	39.153	-.17871

**Table 6: Coefficients of Equation (30)**

Two options are available at this point. First, equation (30) can be smoothed in an expanding averaging process similar to that of equation (23). After sufficient smoothing the value converged to should approach a multiple of the  $\lambda_{amb}$  value in the table. Dividing by the  $\lambda_{amb}$  value and rounding to the nearest whole number will give the integer by which the primary whole cycle ambiguities need to be corrected.

A second option is better, if one wishes to use the refraction corrected carrier-phase measurements before the smoothing has been completed. Specifically, the coefficients in equation (30) (i.e. the values in each row of Table 6) can be scaled by a constant which will cause the value of  $\lambda_{amb}$  to exactly cancel the ambiguity induced error in equation (21) as represented by the final column of Table 5. These biases in Table 5 were obtained by inserting the coefficients computed in Table 4 into equation (21) for a given one cycle ambiguity error. The appropriate scaling and the revised coefficients are given in Table 7. The  $\lambda_{amb}$  value for each row (not shown) is the negative of the final column of Table 5.

Mid. Freq.	Scale	$C_a$	$C_b$	$C_c$
GPS L2	1.18655	15.558	-84.346	68.788
Galileo E6	.727304	18.700	-53.855	35.155
1.3299 GHz	.613668	23.231	-47.258	24.027

**Table 7: Rescaled Coefficients**

When equation (30) is multiplied by the scale factor given in Table 7, the modified coefficients of equation (30) which appear in Table 7 correspond precisely to the difference between equations (11) and (21). This is verified by subtracting the coefficients in Table 4 from the coefficients in Table 2. The difference is the coefficient values given in Table 7, which verifies that, with the appropriate scaling, the offset values of equation (21) can be obtained via alternate means. Substituting the coefficients of Table 7 into equation (31) gives a direct equation which can be used to compute the offset,  $O_s$ .

$$O_s = C_a \Phi_a + C_b \Phi_b + C_c \Phi_c \quad (31)$$

It is appropriate to give at least one more example of how one might compute an alternate value of the scaled offset for subsequent smoothing. As stated above, differencing any two refraction corrected measurements of the range should cancel that range and leave a scaled version of the

offset of equation (22). In this example we will use the refraction-corrected range equation from the first and second frequencies and will subtract from it the refraction-corrected range equation from the second and third frequencies. Specifically, refraction correcting equations (3) and (4) to eliminate the ionospheric effect gives:

$$\frac{f_a^2}{f_a^2 - f_b^2} \Phi_a - \frac{f_b^2}{f_a^2 - f_b^2} \Phi_b = \rho \quad (32)$$

The parallel equation for a refraction corrected range from the  $f_b$  and  $f_c$  frequencies is:

$$\frac{f_b^2}{f_b^2 - f_c^2} \Phi_b - \frac{f_c^2}{f_b^2 - f_c^2} \Phi_c = \rho \quad (33)$$

Subtracting equation (33) from (32) gives the scaled offset equation:

$$O_s = \frac{f_a^2}{f_a^2 - f_b^2} \Phi_a - \frac{f_b^2(f_a^2 - f_c^2)}{(f_a^2 - f_b^2)(f_b^2 - f_c^2)} \Phi_b + \frac{f_c^2}{f_b^2 - f_c^2} \Phi_c \quad (34)$$

The coefficients and wavelength (scale) are evaluated for the three different middle frequencies in Table 8.

Mid Freq.	$C_a$	$C_b$	$C_c$	$\lambda_{amb}$
GPS L2	2.5457	-13.801	11.255	-.0177497
Galileo E6	2.9312	-8.4416	5.5104	-.0170685
1.33 GHz	3.4795	-7.0781	3.5986	-.0164258

**Table 8: Coefficients of Equation (34)**

Like the prior alternative these coefficients can be rescaled to cause the effect of a whole cycle ambiguity error (in each primary phase measurement) to exactly cancel the dependence of the ambiguity induced error in equation (21). This scale factor is obtained by dividing the value of the whole cycle ambiguity in equation (34), i.e. last column of Table 8, into the whole cycle ambiguity in equation 21, i.e. last column of Table 5 (and changing the sign). The resultant scale factor and revised coefficients are given in Table 9.

Mid. Freq.	Scale	$C_a$	$C_b$	$C_c$
GPS L2	6.1116	15.558	-84.346	68.788
Galileo E6	6.3797	18.700	-53.855	35.155
1.33 GHz	6.6766	23.231	-47.258	24.027

**Table 9: Rescaled Coefficients**

Table 9 has exactly the same rescaled coefficients as Table 7 and the offset value,  $O_s$ , is obtained again by inserting these coefficients into equation (31). This shows that the same equation for the offset value,  $O_s$ , results from multiple alternative derivations. The scaled versions of equation (30) and equation (34) are identical.

## CONCLUSIONS

A new three-frequency technique for obtaining geometry free, refraction-corrected, ambiguity-resolved, carrier-phase measurements has been described. First, the ambiguities on at least two wide lane carrier-phase differences are obtained by averaging the corresponding frequency weighted code measurements. These two ambiguity-resolved measurements are then combined into a composite refraction-corrected measurement. The resulting composite is quite noisy due to the amplification of the multipath noise in the original carrier-phase measurements. However, this noisy refraction-corrected carrier phase measurement can be smoothed with another refraction-corrected carrier phase composite measurement constructed to minimize the noise. This later measurement can be constructed from the primary carrier phase measurements prior to resolving their whole cycle ambiguities. By smoothing the difference in the two refraction-corrected measurements, the noise can be reduced and the bias in the low-noise measurement (due to incorrect ambiguities) can be estimated and subsequently corrected.

## REFERENCE

1. Hatch, R.R., US Patent # 6,934,632. "Method for using three GPS frequencies to resolve carrier-phase integer ambiguities," Aug. 23, 2005.
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