A Fast Ambiguity Resolution Technique for RTK Embedded Within a GPS Receiver

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Biography

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Mr. Ron Hatch is the Director of Navigation Systems at NavCom Technology, a company of which he was a co-founder. He has developed a number of innovative techniques for processing GPS measurements and has obtained more than a dozen patents related to GPS. Ron is currently the immediate past president of the Institute of Navigation, is a Fellow of the ION and has received the Kepler and Thurlow awards from the ION.

Abstract

GPS carrier phase integer ambiguity resolution is still the key issue and challenge for high accuracy RTK survey and navigation. Due to speed and memory limitations of OEM GPS receiver boards, the calculation efficiency is of prime importance since it affects the elapsed time before ambiguities can be resolved. Of course, the reliability of the ambiguity resolution process is also a significant concern.

A new technique is presented based upon the Residual Sensitivity Matrix [8], which relates the search integer ambiguity set to each carrier phase residual directly. The technique uses the Singular Value Decomposition of the Residual Sensitivity Matrix to find the minimum search space. The technique not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability. The search space is minimized by selecting only those combinations of possible ambiguity values which are consistent with the satellite geometry and the measurement residuals.

The technique has been implemented within NavCom Technology's NCT 2000D GPS OEM engine. The RTK function embedded within the engine is a background process, which takes about two seconds elapsed time to search a single epoch of data using a 20MHz CPU. Extensive experimental data shows that 85% of the searches yield the correct integer ambiguity resolution using a single epoch of data within a 9 km baseline. Test results in different scenarios are shown in the article.

KEY WORDS: Global Positioning System (GPS), Ambiguity Resolution, Real Time Kinematic (RTK), Residual Sensitivity Matrix, and Singular Value Decomposition.

1 Introduction

Fast integer ambiguity resolution, such as instantaneous ambiguity resolution and ambiguity resolution on-the-fly, is the prerequisite for RTK. Among many specifications of the RTK, three of them are most important. They are ambiguity resolution initialization time, ambiguity resolution reliability, and RTK accuracy. These three are related to each other. For the real-time applications, the ambiguity resolution initial time is important. Lots of effort has been devoted to improving the ambiguity initialization time. Various tests show the RTK algorithm embedded within the NCT2000D GPS engine gets the ambiguity resolved within 2 seconds with 90% probability. More than 99.8% reliability is achieved for the NCT2000D by the integrated voting method to detect and isolate the unique correct candidate set. The accuracy of the RTK depends upon the carrier phase measurement accuracy. The carrier phase resolution of NCT2000D GPS engine is $\frac{1}{256}$ wave length, which is within 0.1 mm and accurate enough for RTK survey and navigation. Also, NCT2000D's patented multipath mitigation and special differential technology make the accuracy of the NCT 2000D RTK to be among the best in the state of the art.

For the RTK embedded within the GPS receiver, the calculating efficiency, which can minimize the data collection interval, is critically important due to the speed limitation of the microprocessor within the NCT2000D engine. For fast ambiguity resolution, searching around the given initial value is needed, as discussed in Section 2. Hence, minimizing the search space will not only make the calculation efficient and shorten the initialization time, but also improve the successful rate of integer ambiguity resolution. Several unique features are designed to minimize the computation tasks. They are briefly described below:

1.1 Base Station Correction Broadcast

The base station transmits corrections rather than the raw data, which most RTK implementations transmit. There are several advantages:

- it offloads part of the computation from the user receiver to the base station receiver;
- it allows code smoothing of the base station data to occur even before the user receiver is turned on, which ensures a more accurate initial code solution;
- it simplifies the processing algorithms because no differencing across receivers is required.

1.2 Combing L1 and L2 Measurements

After differential calculation, the L1 and L2 pseudorange and carrier phase can be written as

$$\nabla\rho_1/\lambda_1 = \frac{f_1}{c}r + \frac{f_2}{c}I_a + \frac{f_1}{c}(MP_1 + \eta_1) \quad (1)$$

$$\nabla\rho_2/\lambda_2 = \frac{f_2}{c}r + \frac{f_1}{c}I_a + \frac{f_2}{c}(MP_2 + \eta_2) \quad (2)$$

$$(\nabla\phi_1 + N_1) = \frac{f_1}{c}r - \frac{f_2}{c}I_a + \frac{f_1}{c}n_1 \quad (3)$$

$$(\nabla\phi_2 + N_2) = \frac{f_2}{c}r - \frac{f_1}{c}I_a + \frac{f_2}{c}n_2 \quad (4)$$

As described in [9], adding eqn. (1) to eqn. (2) and re-arranging yields:

$$\left(\frac{\nabla\rho_1}{\lambda_1} + \frac{\nabla\rho_2}{\lambda_2}\right)\lambda_n = r + I_a + \frac{\lambda_n}{\lambda_1}(MP_1 + \eta_1) + \frac{\lambda_n}{\lambda_2}(MP_2 + \eta_2), \quad (5)$$

with $\lambda_n = \frac{c}{f_1+f_2}$ called narrow lane. subtracting eqn. (4) from eqn. (3) and re-arranging yields:

$$(\nabla\phi_1 - \nabla\phi_2)\lambda_w = r + I_a - (N_1 - N_2)\lambda_w + \frac{\lambda_w}{\lambda_1}n_1 - \frac{\lambda_w}{\lambda_2}n_2. \quad (6)$$

with $\lambda_w = \frac{c}{f_1-f_2}$ called wide lane.

Since the right hand sides of eqn. (5) and eqn. (6) are directly comparable, the difference of the two equations,

$$\left(\frac{\nabla\rho_1}{\lambda_1} + \frac{\nabla\rho_2}{\lambda_2}\right)\lambda_n - (\nabla\phi_1 - \nabla\phi_2)\lambda_w = (N_1 - N_2)\lambda_w + \frac{\lambda_n}{\lambda_1}(MP_1 + \eta_1) + \frac{\lambda_n}{\lambda_2}(MP_2 + \eta_2) - \frac{\lambda_w}{\lambda_1}n_1 + \frac{\lambda_w}{\lambda_2}n_2, \quad (7)$$

provides a basis for estimating the wide lane integer $(N_1 - N_2)$ with the ionospheric delay being canceled, the carrier noise n_1 and n_2 being small, and the coefficients of the code noise ($\frac{\lambda_n}{\lambda_1}$ and $\frac{\lambda_n}{\lambda_2}$) being significantly less than one. The standard deviations of terms of the multipath and noise on each estimate of $(N_1 - N_2)$ produced by eqn. (7) is approximately 0.7 times those of L1 or L2. The correct integer ambiguity of the wide-lane phase formed by eqn. (7) can be solved more easily than that of L1 or L2. Once $(N_1 - N_2)$ is determined, eqn. (6) is available for accurate positioning with the wide-lane carrier phase and for aiding in direct estimation of N_1 and N_2.

1.3 Phase Smoothed Code across Measurements' samples

Given the GPS code and carrier phase measurements, it is natural to consider the methods of combining two measurements to achieve the higher-accuracy range information for the integer ambiguity round off. One method is to regard the integer ambiguity as a real number, then try to solve the ambiguity as a part of the stochastic estimation process, which is summarized in Section 7.4.1 and Section of 7.4.2 of [3] and needs more computation for both the dynamic updating and covariance calculation. Another method is a decoupled implementation, which employs the Hatch filter [9] and is more efficient. Both methods can be referred to as carrier-smoothed-code techniques.

For the RTK embedded within NCT2000D engine, the decoupled phase-smoothed-code is implemented by averaging the measurements based on the first row of eqn. (7), which can recursively reduced the multipath and noise errors across the measurement samples.

1.4 Residual Sensitive Matrix

The residual Sensitive Matrix (S-Matrix), defined and derived in [8] and re-written in Section 3.1, relates the integer ambiguity set to the quadratic measurement residual vector directly for the unique correct integer ambiguity set validation and detection. It is computationally efficient, in that a simple process is used to compute the residual vector without computing a position solution or the related adjustments. Position solutions are computed only when the residual vector indicates the particular integer ambiguity permutation to be the unique correct set via the integrated voting method developed by NavCom Technology, Inc.

This article details the new improvement based on the Singular Value Decomposition (SVD) of the Residual Sensitivity Matrix to find the minimum search space. The technique not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability. Before going into the new technique, let us briefly review the integer ambiguity resolution.

2 Background of the Integer Ambiguity Resolution

GPS Integer ambiguity resolution is the key step to utilize carrier phase measurements for high accuracy navigation. The objective of the GPS integer ambiguity resolution is to solve for the unknown number of integer carrier cycles biasing the phase measurement, so that the low noise carrier phase measurement can be used as a range signal.

There are various categories of applications that need GPS integer ambiguity resolution:

- Short range vs. long range;
- Static position vs. dynamic/kinematic position;
- Real-time processing vs. post-processing;
- Known or unknown baseline.

Differential carrier phase can be written as

$$\nabla\phi\lambda = \left[\begin{array}{cc} \mathbf{h} & \lambda \end{array}\right] \left[\begin{array}{c} \mathbf{x} \\ N \end{array}\right] + n_\phi \tag{8}$$

with $\nabla\phi$ being the differential carrier phase, $\mathbf{h}$ the vector between the antenna of NCT2000D and the satellites, $\mathbf{x}$ being the linearized position, N being the integer ambiguity, and n_ϕ being the differential phase noise plus the multipath error.

Eqn. (8) can not be solved directly since there are $(4+n)$ unknown variables for n single difference GPS carrier phase measurements and $(3+n)$ unknown variables for n double difference GPS carrier phase measurements. Theoretically, eqn. (8) cannot be used directly as a linear measurement model to estimate the real vector $\mathbf{x}$ and the integer ambiguity N by common methods, such as the Least Square and the Kalman filter, due to the fact that N is an integer. Hence, the estimation problem, based on eqn. (8), is nonlinear.

Therefore, in practical applications, GPS integer ambiguity resolution is accomplished by

- **solving** the integer ambiguity with special search and hypothesis testing techniques, and
- **validating** the result to make sure the integer ambiguity solution is unique and correct.

To solve and validate the integer ambiguity, there are three categories of methods. They are:

- Long duration static observation [12]: This method is used in static mode where x is unchanged. The reasons for long term observation are that long time observation data can average the multipath error and GPS receiver noise and that due to h varying slowing (caused by GPS satellite motion) approximately 20 minutes are required for h to change enough for the set of equations to yield observability. The long convergence time is the major drawback of this approach.
- GPS antenna special moving: GPS antennae swapping is described in [11]. Swapping location of two antennae causes the observability properties of the h matrix to change rapidly, but is rarely possible in real time kinematic and long baseline situations.
- Searching methods: These methods need few assumptions and have attracted the attention of many researchers. Numerous methods have been reported: Ambiguity Function Method (AFM) [2], Fast Ambiguity Resolution Approach (FARA) [4], Least Squares Ambiguity Search Technique [7, 3], Cholesky Decomposition [10], Fast Ambiguity Search Filter (FASF) [1], Least Square AMBiguity Decorrelation Adjustment (LAMBDA) [13], and Integrated Ambiguity Resolution Method [6].

The first two categories of methods are straightforward. The basic theory and steps of GPS integer ambiguity resolution search methods are (see [14] for details):

- **Linear stochastic model definition**: Both differential pseudorange and carrier phase measurements have the linear equation as eqn. (8).
- **Ambiguity resolution initialization:** This defines the initial integer ambiguity set and its search range.
- **Search space reduction:** The initial integer ambiguity, initial real states, and their search ranges define the whole search space. For instance, n differential GPS measurements form a n dimension integer search space.
- **State and standard deviation calculation in the reduced search space:** For each integer ambiguity candidate set in the reduced space, estimate the real states and calculate the residual and standard deviation to identify the unique ambiguity candidate set.

- **Validation and rejection criteria for the unique and correct candidate:** The statistic test is based on the statistical hypothesis testing theory.

Among these issues, search space reduction is critically important. It not only affects the ambiguity resolution speed, but also defines the ambiguity resolution success rate. The smaller the search space, the easier to find the unique and correct candidate set.

One approach, to reduce the number of integer candidates without missing the correct candidate, is to decrease the off-diagonal element values of the covariance matrix that determine the search range. Two terms affect the covariance matrix. They are H, determined by GPS satellite geometry, and R, formed by the measurement residue errors. GPS users can do nothing about the satellite geometry. Hence, the existing methods to decrease the covariance matrix value are: improving the GPS receiver measurement performance; combining L1 measurement and L2 measurement to suppress the pseudorange measurement noise for dual frequency GPS receiver, or using phase smooth code (such as, Hatch filter) to reduce the measurement noise; and, applying the integer inverse matrix transformation to decorrelate the double differential measurement and to reduce the integer ambiguity covariance element value [11].

Second approach is to increase the search step length by using a longer wavelength (combine L1 phase and L2 phase). For the same search space defined in meters, the longer the wavelength, the smaller the number of integer search candidates. However, the phase estimate of the long wavelength usually also amplifies the measurement noise.

Also, the search space can be greatly reduced by cutting the search dimensions. In most cases, the number of satellite in view is more than six. Cutting the search dimensions can greatly reduce the number of searching candidates. For example, if each search range of four integer ambiguities is 10 cycles, the total search ambiguity candidate set size is 10^4, while for seven integer ambiguities, the total search ambiguity candidate set size is 10^7. The idea to partition the differential GPS measurements into a primary measurement set and a secondary measurement set was first presented in [7]. The phase measurements of the primary set define the reduced search space, while the phase measurements of the secondary set are used to validate the candidates.

3 New Algorithm of the Integer Ambiguity Resolution

This section describes the key idea and the basic equations. The related issues are discussed. The technique is focused on reducing the search space based upon the S matrix defined in [8].

3.1 Ambiguity Search Space Definition and Validation Criteria

For each satellite, the differential measurement eqn.(8) can be re-written as:

$$(\nabla\phi + N)\lambda = \mathbf{h}\mathbf{x} + n_\phi \tag{9}$$

If there are n satellites in the view, all the measurements can be written in array format as:

$$(\nabla\mathbf{\Phi} + \mathbf{N})\lambda = \mathbf{H}\mathbf{x} + \mathbf{n}_\phi \tag{10}$$

where $\nabla\mathbf{\Phi} = [\nabla\phi_\mathbf{1}, \nabla\phi_\mathbf{2}, \cdots, \nabla\phi_\mathbf{n}]^T$ is the differential carrier phase measurement vector formed by each satellite, $\mathbf{N} = [N_1, N_2, \cdots, N_n]^T$ is integer ambiguity vector formed by each satellite, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_n]^T$ is the measurement vector matrix from user to satellites with $\mathbf{h}_i$ being the $\mathbf{h}$ of ith satellite, and $\mathbf{n}_\phi = [n_{\phi_1}, n_{\phi_2}, \cdots, n_{\phi_n}]^T$ is the carrier phase measurement noise vector formed by each satellite.

Using the pseudorange or carrier phase smoothed pseudorange, the initial ambiguity $\hat{\mathbf{N}}_\mathbf{0}$ can be estimated by rounding off. If the search width of each satellite is δN, the total candidate set number is δN^{n-1} with n being the number of satellites used. For instance, if $\delta N = 4$ and $n = 7$, the total search number is $4^6 = 4096$. And for each candidate set, the real state is

$$\hat{\mathbf{x}} = [\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^T\mathbf{R}^{-1}(\nabla\mathbf{\Phi} + \hat{\mathbf{N}}_\mathbf{0} + \mathbf{\Delta N})\lambda \tag{11}$$

where $\mathbf{R} = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{bmatrix}$ is the measurement covariance matrix formed by the differential carrier phase noise with σ_i being the standard deviation of satellite i differential carrier phase noise, $\mathbf{\Delta N} = [\delta N_1, \delta N_2, \cdot, \delta N_n]^T$ is integer ambiguity vector formed from search width δN for each satellite.

The calculated phase range residual vector is:

$$\begin{aligned}\Delta_\Phi &= (\nabla\Phi + \hat{\mathbf{N}}_\mathbf{0} + \mathbf{\Delta N})\lambda - \mathbf{H}\hat{\mathbf{x}} &(12)\\ &= (\mathbf{I} - \mathbf{H}[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^T\mathbf{R}^{-1})(\nabla\Phi + \hat{\mathbf{N}}_\mathbf{0} + \mathbf{\Delta N})\lambda \\ &= \mathbf{S}(\nabla\Phi + \hat{\mathbf{N}}_\mathbf{0} + \mathbf{\Delta N})\lambda &(13)\end{aligned}$$

with

$$\mathbf{S} = \mathbf{I} - \mathbf{H}[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^T\mathbf{R}^{-1} \tag{14}$$

which was defined in [8]. The estimated phase standard deviation for candidate set $\hat{\mathbf{N}}$ is:

$$\sigma_{\Phi|\hat{\mathbf{N}}} = \sqrt{\frac{\Delta_\Phi^T\Delta_\Phi}{n-k}} \tag{15}$$

with k being the real state number of $\mathbf{x}$ ($k = 4$ for single differential GPS and $k = 3$ for double differential GPS).

The target of the ambiguity search is to find the unique and correct candidate set with smallest $\sigma_{\Phi|\hat{\mathbf{N}}}$. Since Δ_Φ is a vector, minimizing $\sigma_{\Phi|\hat{\mathbf{N}}}$ is equal to minimize the absolute value of each term of Δ_Φ.

3.2 Property of S Matrix

The $\mathbf{S}$ matrix has many nice properties, such as symmetric, zero sum of each row and column ($\sum_{i=1}^{n} s_{ij} = 0$ and $\sum_{j=1}^{n} s_{ij} = 0$), and positive semidefinite. The following properties are useful for the search space reduction derivation described below:

1. **Equal idempotent**: $\mathbf{S} = \mathbf{S^2} = \mathbf{S^3} = \cdots\cdots$;

2. **Rank equal to** n-k: $rank(\mathbf{S}) = n - k$ (with k=4 for single differential GPS and k=3 for double differential GPS);

3. **SVD calculation efficiency of S matrix**: For SVD of $\mathbf{S} = \mathbf{UXV^T}$, one of the solution of $\mathbf{V}$ is equal to the eigenvector of $\mathbf{S}$. The eigenvalue of $\mathbf{S}$ is either 1 or 0, and its eigenvectors are all real.

It is easy to show the above properties. To make the statement clear without being bothered by mathematical derivation, they are omitted here.

3.3 Derivation Based on Singular Value Decomposition for Search Space Reduction

The initial phase range residual vector

$$\Delta_{\Phi_0} = \mathbf{S}(\nabla\Phi + \hat{\mathbf{N}}_\mathbf{0})\lambda, \tag{16}$$

minimizing the absolute value of Δ_Φ in eqn. (13) is equal to estimate $\Delta\mathbf{N}$ that

$$\Delta_{\Phi_0} + \mathbf{S}\boldsymbol{\Delta}\mathbf{N}\lambda = \mathbf{0} \qquad \Rightarrow \qquad \mathbf{S}\boldsymbol{\Delta}\mathbf{N} = -\Delta_{\Phi_0}\frac{1}{\lambda} = -\mathbf{r}_0 \tag{17}$$

with $\mathbf{r}_0$ being the initial phase range residual vector in unit of cycle.

Since $\mathbf{S}$ is not full rank, $\mathbf{S}$ can be re-written by using Singular Value Decomposition (SVD) as

$$\mathbf{S} = \mathbf{UXV^T} \tag{18}$$

where

$$\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2 \cdots \mathbf{u}_n] \tag{19}$$

with

$$\mathbf{u}_i{}^T\mathbf{u}_i = 1, \qquad \mathbf{u}_i{}^T\mathbf{u}_j = 0 \ \ (i \neq j)$$

with $\mathbf{u}_i$ being orthonormal vectors and $\mathbf{U}$ having full rank n;

$$\mathbf{X} = \begin{bmatrix} \begin{matrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{n-k} \end{matrix} & \mathbf{0}_{(n-k)\times k} \\ \mathbf{0}_{k\times(n-k)} & \mathbf{0}_{k\times k} \end{bmatrix} \tag{20}$$

with $s_1 = \cdots = s_{n-k} = 1$ for matrix $\mathbf{S}$; and

$$\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_n] \tag{21}$$

with

$$\mathbf{v}_i{}^T\mathbf{v}_i = 1, \qquad \mathbf{v}_i{}^T\mathbf{v}_j = 0 \ \ (i \neq j)$$

with $\mathbf{v}_i$ being orthonormal vectors and $\mathbf{V}$ having full rank n.

To estimate $\mathbf{S}$ that $\mathbf{S}\boldsymbol{\Delta}\mathbf{N} = -\mathbf{r}_0$, we have

$$\begin{aligned} & \mathbf{UXV^T}\boldsymbol{\Delta}\mathbf{N} = -\mathbf{r}_0 \\ \Leftrightarrow \quad & \mathbf{XV^T}\boldsymbol{\Delta}\mathbf{N} = -\mathbf{U}'\mathbf{r}_0 = \mathbf{r}_1 \\ \Leftrightarrow \quad & \begin{bmatrix} s_1\mathbf{v}_1^T \\ \vdots \\ s_{n-k}\mathbf{v}_{n-k}^T \\ \mathbf{0}_{k\times n} \end{bmatrix} \boldsymbol{\Delta}\mathbf{N} = \mathbf{r}_1. \end{aligned} \tag{22}$$

Eqn. (22) can be re-written as

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{0}_{k\times(n-k)} & \mathbf{0}_{k\times k} \end{bmatrix} \begin{bmatrix} \mathbf{N}_{1_f} \\ \mathbf{N}_{2_f} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{11} \\ \mathbf{r}_{12} \end{bmatrix} \tag{23}$$

$$\Leftrightarrow \quad \mathbf{A}_1\mathbf{N}_{1_f} + \mathbf{A}_2\mathbf{N}_{2_f} = \mathbf{r}_{11}$$

$$\Leftrightarrow \quad \mathbf{N}_{1_f} = \mathbf{A}_1^{-1}(\mathbf{r}_{11} - \mathbf{A}_2\mathbf{N}_{2_f}) \tag{24}$$

$$\Leftrightarrow \quad \mathbf{N}_1 = round(\mathbf{A}_1^{-1}(\mathbf{r}_{11} - \mathbf{A}_2\mathbf{N}_2))$$

$$\Leftrightarrow \quad \mathbf{N}_1 = round(\mathbf{C} - \mathbf{D}\mathbf{N}_2) \tag{25}$$

with $\mathbf{r}_{12} = \mathbf{0}$, $\mathbf{C} = \mathbf{A}_1^{-1}\mathbf{r}_{11}$, and $\mathbf{D} = \mathbf{A}_1^{-1}\mathbf{A}_2$.

Eqn. (25) gives the relationship of two integer ambiguity subsets, which will reduce the search space from searching around n satellites for both $\mathbf{N}_1$ and $\mathbf{N}_2$ to searching around k satellites for $\mathbf{N}_2$ only.

After calculating the integer $\mathbf{N}_1$ and $\mathbf{N}_2$, substituting $\mathbf{N}_1$ and $\mathbf{N}_2$ into eqn. (23) yields

$$\mathbf{r} = \mathbf{r}_{11} - (\mathbf{A}_1\mathbf{N}_1 + \mathbf{A}_2\mathbf{N}_2), \tag{26}$$

which is the residual corresponding to integer set $\mathbf{N}_1$ and $\mathbf{N}_2$.

It can be shown that

- $norm(\mathbf{r})$=$norm(\mathbf{r}_{11})$ and minimizing the norm value of $\mathbf{r}$ will minimize the norm value of $\mathbf{r}_0$.

- $\mathbf{N}_2$ is formed by k satellites ($k = 4$ for single differential GPS, $k = 3$ for double differential GPS). However, searching around three satellites is enough in the whole search space for both single differential GPS and double differential GPS.

3.4 Further Calculation Improvement for Implementation

Though the calculation of SVD is efficient, a better way for implementation is shown below. Multiplying $\mathbf{S}$ on both side of eqn. (17) and applying Property 1 of **S** matrix yield

$$\mathbf{S\Delta N} = -\mathbf{Sr}_0 \quad \Rightarrow \quad \mathbf{S}(\mathbf{\Delta N} + \mathbf{r}_0) = \mathbf{0}. \tag{27}$$

From Property 2 of **S** matrix, there are $n-k$ rows of **S** matrix that are linearly independent. Selecting $n-k$ rows and rearranging the **S** with first $n-k$ rows independent, eqn. (27) can be rewritten as

$$\begin{bmatrix} \mathbf{S_{11}} & \mathbf{S_{12}} \\ \mathbf{S_{21}} & \mathbf{S_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta N_1} + \mathbf{r_{01}} \\ \mathbf{\Delta N_2} + \mathbf{r_{02}} \end{bmatrix} = \mathbf{0} \tag{28}$$

with $\mathbf{S_{11}} \in \Re^{(n-k)\times(n-k)}$, $\mathbf{S_{12}} \in \Re^{(n-k)\times k}$, $\mathbf{S_{21}} \in \Re^{k\times(n-k)}$, $\mathbf{S_{22}} \in \Re^{k\times k}$, $\mathbf{\Delta N_1} \in \Re^{(n-k)\times 1}$, and $\mathbf{\Delta N_2} \in \Re^{k\times 1}$.

From eqn. (28), we have

$$\mathbf{S_{11}}(\mathbf{\Delta N_1} + \mathbf{r_{01}}) + \mathbf{S_{12}}(\mathbf{\Delta N_2} + \mathbf{r_{02}}) = \mathbf{0} \tag{29}$$

$$\Rightarrow \quad \mathbf{\Delta N_{1f}} = -\mathbf{T}(\mathbf{\Delta N_2} + \mathbf{r_{02}}) - \mathbf{r_{01}} \tag{30}$$

with

$$\mathbf{T} = \mathbf{S_{11}}^{-1}\mathbf{S_{12}}. \tag{31}$$

Hence, the ambiguity search space is formed by $\mathbf{\Delta N_2}$ subset, while $\mathbf{\Delta N_1}$ can be calculated based on eqn. (30) and its integer value is

$$\mathbf{\Delta N_1} \quad = \quad round(\mathbf{\Delta N_{1f}}). \tag{32}$$

The corresponding residual of each candidate set, formed by searching around $\mathbf{\Delta N_2}$ and calculating $\mathbf{\Delta N_1}$ directly, is

$$\Delta_{\Phi_1} \quad = \quad \mathbf{\Delta N_1} - \mathbf{\Delta N_{1f}} \tag{33}$$

Hence, the residual vector for all the satellites is

$$\Delta_\Phi \quad = \quad \mathbf{S_1}\Delta_{\Phi_1} \tag{34}$$

with

$$\mathbf{S_1} = \begin{bmatrix} \mathbf{S_{11}} \\ \mathbf{S_{21}} \end{bmatrix}. \tag{35}$$

The corresponding phase standard deviation for the candidate set can be calculated by eqn. (15), which is used to decide whether is the correct and unique integer candidate set.

3.5 Satellite Selection

The criteria for selecting the satellites to search around is of interest. The objective is to minimize the measurement error (composed by carrier phase noise and multipath error) that affect the estimate of $\mathbf{N_1}$. The measurement error is indicated in **T**. Therefore, the target is to select the satellites that minimize the element value of matrix $\mathbf{S_{11}}^{-1}$ in $(n-k) \times (n-k)$ dimensions to decrease the measurement error effect for **N** estimation. This is easy to define, since $\mathbf{S_{11}}$ is always positive definite. One of the sub-optimal solutions is to find the satellite combination that can maximize the norm of $\mathbf{S_{11}}$, which will minimize the element value of the **T**.

Baseline Type	Resulted Num.	Correct Num.	Wrong Num.	Error Rate
70 m	28255	28255	0	0.00%
9 km	30391	30331	60	0.04%

Table 1: GPS integer ambiguity resulting rates

Baseline Type	Resulted Num.	1 Epoch Num.	1 Epoch Rate
70 m	28255	23163	81.98%
9 km	30391	25796	84.88%

Table 2: GPS Integer ambiguity 1 epoch data resulting rates

3.6 Calculation Efficiency and Comparison with Existing Method

The approach applies for both L_1, L_2, L_w, and L_n. For each search epoch, the approach needs to calculate **S** matrix, split **S** matrix, and calculate **T** matrix once. The total candidate set number of this approach is N^3. For example, if $n = 7$, $m = 3$, and $\delta N = 4$, the total search number is less than 64.

4 Experimental Results

The RTK ambiguity search process described above has been incorporated into NCT2000D GPS OEM engine. Also, the DGPS integer ambiguity resolution technique described above has been implemented in a post-processing program. The program allows one to perform the logged data set for the ambiguity search continuously. As soon as one set of integer ambiguity candidate set is declared correct, the search process is re-initialized and a new search will start right away.

Figure 1, Table 1, Table 2 , and Table 3 show the search results of 70 meter short baseline for 21.88 hours. The total resulting number is 28255 with 28255 being correct (the difference between the truth position and position calculated by ambiguity solved carrier phase measurements is less than the half of the wave length) and 0 error. Hence, the error rate is 0. The average search time is 1.789 seconds. and there are 23163 search runs finished by using only one epoch data measurement, which means

Baseline Type	First Max. Search Time	Second Max. Search Time	Third Max. Search Time
70 m	72	68	55
9 km	175	145	117

Table 3: The first three maximum search time in seconds

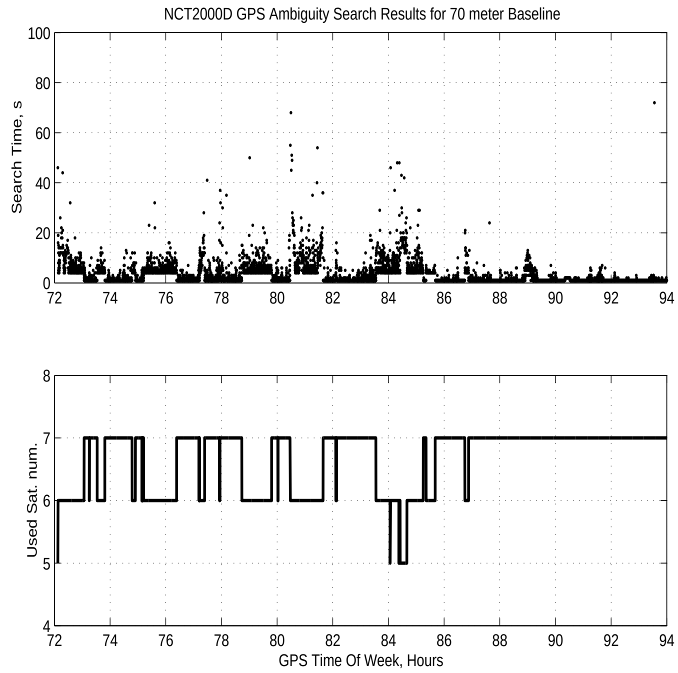

Figure 1: NCT2000D GPS ambiguity results for 70m baseline

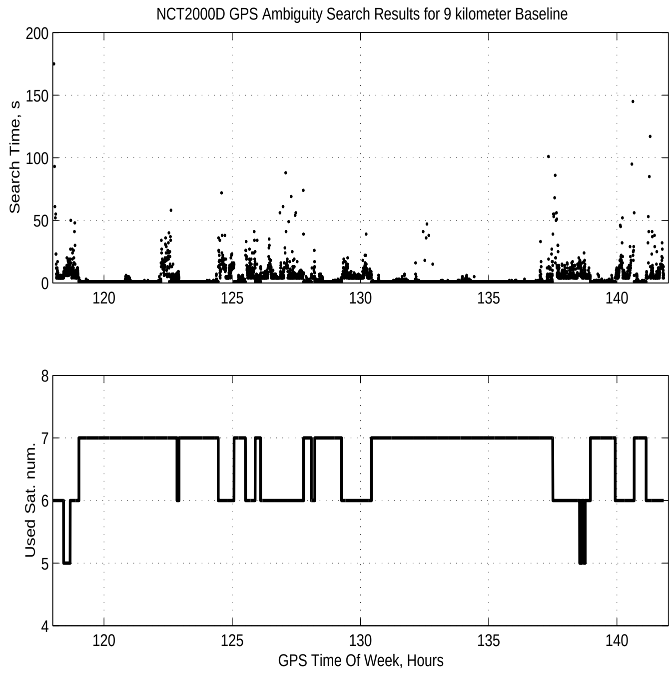

Figure 2: NCT2000D GPS ambiguity results for 9km baseline

the instantaneous ambiguity resolution rate is 81.98%. The standard deviation of the position error with respect to the truth is 0.012 meters. The first maximum search time is 72 seconds, the second 68 seconds, and the third 55 seconds.

Figure 2, Table 1,, Table 2 , and Table 3 show the search results of 9 kilometer short baseline for 24 hours. The total resulting number is 30391 with 30331 being correct (the difference between the truth position and position calculated by ambiguity solved carrier phase measurements is less than the half of the wave length) and 60 error. Hence, the error rate is less than 0.2%. The average search time is 1.848 seconds. and there are 25796 search runs finished by using only one epoch data measurement, which means the instantaneous ambiguity resolution rate is 84.88%. The standard deviation of the position error with respect to the truth is 0.029 meters. The first maximum search time is 175 seconds, the second 145 seconds, and the third 117 seconds.

5 Conclusion

For the RTK embedded within the GPS engine, the calculating efficiency, which can minimize the data collection interval, is critically important due to the speed limitation the microprocessor. A new technique, which minimizes the search space and relates the search integer ambiguity set directly to carrier phase residual vector, is presented for calculation efficiency. The technique uses the Singular Value Decomposition of the Residual Sensitivity Matrix to find the minimum search space. The technique not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability.

Experimental results with the technique embedded in NCT 2000D GPS OEM engine shows the RTK search speed is improved significantly. Extensive experimental data shows that 90% of the searches yield the correct integer ambiguity resolution within two data epoch for a 9 km baseline.

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