

A COMPUTATIONALLY EFFICIENT AMBIGUITY RESOLUTION TECHNIQUE

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BIOGRAPHY

Ron Hatch is the Director of Navigation Systems at NavCom Technology, a company of which he was a co-founder. He has developed a number of innovative techniques for processing GPS measurements and has obtained more than a dozen patents related to GPS. Ron is currently the president of the Institute of Navigation, is a Fellow of the ION and has received the Kepler and Thurlow awards from the ION.

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ABSTRACT

A new method of carrier-phase ambiguity resolution is described. The new technique is a variation of the least-squares residual search technique in the ambiguity domain. It uses a very efficient algorithm to compute the residuals associated with each potential combination of ambiguities to be tested. Several other techniques are employed to simplify the calculations and to enhance the probability of identifying the correct ambiguity vector. The intent is to minimize the number of data epochs required to correctly identify the integer ambiguity values.

The capability of the technique to rapidly identify the correct ambiguity vector is illustrated by plotting the results of scoring runs which exercise the search algorithms using recorded field data taken over different rover-to-base separation distances. To maximize the number of searches exercised in these runs, as soon as the ambiguity vector is identified, the results are cleared and a new search is initiated with all navigation parameters re-initialized. Some navigation results are also shown which are typical of RTK carrier-phase navigation results.

INTRODUCTION

The RTK ambiguity resolution technique implemented within the NavCom dual-frequency receiver has several unique features designed to minimize the computational task while ensuring that a minimal data collection interval is required. The fundamental approach is similar to several least-squares residual search techniques which have been employed by others. [1-4]

The first unique feature is that the base station transmits corrections rather than the raw data, which most RTK implementations transmit. This has several advantages: 1) it offloads part of the computation from the user receiver to the base station receiver; 2) it allows code smoothing of the base station data to occur even before the user receiver is turned on, which ensures a more accurate initial code solution; 3) it simplifies the processing algorithms because no differencing across receivers is required.

The second unique feature is a simplified computational technique to generate the residuals resulting from candidate ambiguity vectors. This technique is the main subject of the paper and will be explored in some detail. The search process is a two-stage process which depends upon the availability of dual-frequency measurements at both the base-station receiver and the user receiver. First a wide-lane search is performed and up to 10 ambiguity vectors, which meet specific requirements (e.g. residuals less than a threshold value) are saved for further processing. In the second stage, each of the ambiguity vectors determined in the first stage are tested by searching each satellite (except the first to avoid redundant clock solutions) across the two narrow-lane ambiguity values which result in a narrow-lane measured range closest to the corresponding wide-lane measured range. Those narrow-lane combinations, which meet a number of specific criteria, are scored and if more than one are acceptable their relative score is used to determine whether or not one of them can be declared the correct ambiguity vector.

INITIAL CODE SOLUTION

The primary motivation in the initial code solution is to obtain the most accurate code solution possible with as few measurement epochs as possible. Since the baseline distances over which the ambiguities can be easily resolved are typically less than 15 kilometers, it is assumed that the ionospheric refraction effects can be modeled at both base station and user with sufficient accuracy that the residual ionospheric errors are almost negligible. This means that the largest errors, which will affect the differential code solution, will be the multipath effects upon the dual-frequency code measurements. The combination of code measurements which has the minimum multipath error (or at least very close to the minimum) is the frequency-weighted sum of the code measurements. This combination also has the advantage that it is exactly matched in ionospheric refraction effects by the wide-lane difference carrier-phase measurement. This means that it can be smoothed by the wide-lane carrier-phase measurement with an increasing time constant, i.e. the multipath error will become smaller and smaller as the satellite is continuously tracked.

Because the base-station receiver is typically put in operation long before the user receiver, our choice of generating the code and carrier phase corrections at the base-station receiver allows the base-station code measurements to be smoothed such that most of the multipath error is removed before the code corrections are generated. This means that the multipath noise contributed by the base station receiver is largely eliminated. This represents a substantial advantage over the normal practice of sending raw base-station measurements to the user.

The frequency weighted combined measurement is:

$$P = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2} \quad (1)$$

where f represents the frequency and P the pseudorange code measurement.

The carrier-phase measurement which is affected by the same amount of ionospheric refraction error is:

$$C = (N_w + \phi_1 - \phi_2)\lambda_w \quad (2)$$

where N is the integer ambiguity, ϕ is the carrier-phase measurement and λ the wavelength.

By setting equation (1) equal to equation (2) and solving for the wide-lane ambiguity, N_w , a new value for the floating ambiguity can be computed each epoch.

$$N_w = \frac{f_1 P_1 + f_2 P_2}{(f_1 + f_2)\lambda_w} - (\phi_1 - \phi_2) \quad (3)$$

This value for the wide-lane floating ambiguity value can be improved each epoch by computing the recursive average over all epochs up to the present time. This smoothed value of N_w can then be substituted back into equation (2) to give a smoothed carrier-phase pseudorange:

$$\bar{C} = (\bar{N}_w + \phi_1 - \phi_2)\lambda_w \quad (4)$$

One can use the process represented by equations (1) through (4) at both the base-station receiver and at the user receiver. At the base station the smoothed carrier-phase pseudorange is differenced with the true range to yield a pseudorange correction which, after removing the common clock effect, is sent to the user receiver. At the user receiver the correction is applied to the combined frequency-weighted code measurement as represented by equation (1). A wide-lane carrier-phase correction is also generated at the base station by arbitrarily selecting a whole-lane integer in equation (2) and differencing it with the true range. The whole-lane error, which is made by arbitrarily selecting the ambiguity value at the base station is absorbed into the whole-lane ambiguity value at the user receiver. After clock removal, this carrier-phase correction is sent to the user receiver and applied to the wide-lane carrier-phase measurement as represented by equation (2). The significant advantage to this process, as indicated above, is that, due to the separate smoothing, the multipath corruption of the base station measurements is usually largely removed even before the user receiver is turned on. This means that the initial smoothed code measurements at the user receiver are more accurate and a more accurate code solution will be obtained.

At the user receiver, the corrected and smoothed value of equation (4) for each satellite is now used in a least-squares process to obtain an initial code solution.

WIDE-LANE SEARCH

While the process described above, i.e. equation (4), gave us a set of floating (non-integer) ambiguity values for each satellite, a better set (fewer degrees of freedom) can be obtained from the initial code solution. Also we note that given a set of integer ambiguities, a second set, differing from the first by a constant integer value for each satellite, will result in the same identical spatial position but with a different clock value. Because our clock solution is of no interest and can be selected arbitrarily, this means that we can constrain one of the ambiguity values arbitrarily. In our process, we choose to use the highest elevation satellite as the reference satellite and force its ambiguity value to zero. Using the initial code position solution, we can compute a range, R , to each satellite and set it equal to the measurement represented by equation (2). Solving the result for the ambiguity value and subtracting the ambiguity value of the highest elevation satellite gives:

$$N_w^i = (R^i - R^r) / \lambda_w - (\phi_1^i - \phi_2^i) + (\phi_1^r - \phi_2^r) \quad (5)$$

where the superscript, i, represents each satellite in turn and the superscript, r, represents the reference satellite—in this case the highest elevation satellite.

Our initial wide-lane carrier-phase RTK solution involves rounding the floating ambiguities so obtained to their integer values, substituting these values into equation (4) and computing a new least-squares position solution. The floating ambiguity given by equation (5) is used in the search process to sequentially select the closest integer values over the selected search range.

We have found that searching over the four closest values is almost always sufficient to include the true ambiguity value. Given n satellites this means that there will be 4^{n-1} permutations of the ambiguity vector. Thus, there will be 1024 permutations when six satellites meet the appropriate elevation criterion and 4096 when seven satellites are available. With each ambiguity permutation a set of wide-lane carrier-phase measurement residuals can be computed. It is these residuals which play a prominent role in determining the correct set of integer ambiguity values. However, obtaining the residuals by re-computing a least-squares solution for each permutation is computationally prohibitive. A much faster way to compute the residuals is to use a residual sensitivity matrix, S .

RESIDUAL SENSITIVITY MATRIX

Given the linearized measurement equation:

$$Hx = z \quad (6)$$

where H is the sensitivity matrix (i.e. the direction cosines) of the state vector (position and clock corrections), x , to the innovations (difference between the measurements and their expected value), z .

Equation (6) can be expanded to include a set of position and clock corrections corresponding to a set of innovations:

$$HX = Z \quad (7)$$

Now if we want to see the separate effect on the position of a single whole-cycle change in the ambiguity value of each satellite, Z will become the identity matrix (or depending on units, the identity matrix times the scalar wavelength).

$$HX = I \quad (8)$$

The least-squares solution for X is then:

$$X = (H^T H)^{-1} H^T \quad (9)$$

For a weighted least-squares solution this becomes:

$$X = (H^T R^{-1} H)^{-1} H^T R^{-1} \quad (10)$$

where R is the measurement covariance matrix.

Note that the left hand side of equation (9) or (10) can be pre-computed and when multiplied by the appropriate column of the identity matrix (or scaled identity matrix) gives the appropriate column of X , which is the associated change in the position and clock caused by the integer ambiguity change.

Multiplying equation (9) by H tells us how much that change in position will affect the innovations.

$$HX = H(H^T H)^{-1} H^T \quad (11)$$

Now if we subtract this change in the innovations from the input value of the innovations (I) we get the effect on the residuals of a whole cycle change in the ambiguity value for each satellite. This is called the residual sensitivity matrix, S , and is given by:

$$S = I - H(H^T H)^{-1} H^T \quad (12)$$

The residual sensitivity matrix, S , for a weighted least squares solution is:

$$S = I - H(H^T R^{-1} H)^{-1} H^T R^{-1} \quad (13)$$

The S matrix has a number of interesting properties. It is symmetric. It is idempotent, i.e. $S=S^2=S^3=\dots$. The sum of any row or column equals zero, i.e. residuals are zero mean. The length of any row or column is equal to the square root of the associated diagonal element. Since the solution vector, x , has four elements, the rank of S is $n-4$ where n is the number of satellites.

The residuals of the initial RTK solution, described in the prior section are updated by adding the product of the S matrix and the matrix formed by scaling the identity matrix diagonal elements by the specific permutation of changes in the integer ambiguity values to be tested. The ten permutations with the smallest root-sum-square (rss) of residuals are saved for further narrow-lane processing if their rss residuals is less than an acceptable threshold value.

STEPPING TO THE NARROW-LANE

Assuming the differential ionospheric refraction is negligible allows us to equate the narrow-lane range to the wide-lane range. Thus:

$$(N_1 + \phi_1) \lambda_1 = (N_w + \phi_1 - \phi_2) \lambda_w \quad (14)$$

Solving equation (14) for the L_1 ambiguity value gives:

$$N_1 = 4 \frac{9}{17} N_w + (3 \frac{9}{17} \phi_1 - 4 \frac{9}{17} \phi_2) \quad (15)$$

It is interesting to note that the deviation from an integer value obtained for N_1 in equation (15) is identical in

information content to the deviation one will obtain in attempting to step to any other narrow or wide-lane ambiguity value. While the deviation is not always the same, it is either the fractional complement or a multiple of the same fraction. Thus to step to N_2 one obtains the analogous equation:

$$N_2 = 3 \frac{9}{17} N_w + (3 \frac{9}{17} \phi_1 - 4 \frac{9}{17} \phi_2) \quad (16)$$

Similarly, the equation to compute the ambiguity value for the average of the carrier-phase on L_1 and L_2 (which typically gives the most accurate position) is:

$$N_A = 4 \frac{1}{34} N_w + (3 \frac{9}{17} \phi_1 - 4 \frac{9}{17} \phi_2) \quad (17)$$

Obviously there is no noise present in the value for N_w in any of the equations above. The noise in the computed value of the integer ambiguity arises from the amplification of the multipath corruption of the carrier-phase measurements in the right-hand term of the above equations. This noise rarely exceeds one-cycle. Thus, a search for the narrow-lane ambiguity value which minimizes the rss residuals, need only test for the two integer values closest to the value computed in the above equations.

SCORING THE INTEGER COMBINATIONS

The wide-lane integer combinations with rss residuals less than a threshold, or if more than ten, the ten combinations with the smallest rss residuals are subject to the narrow-lane search and assigned a score. Like a golf score, the lowest scores are considered the best scores. The score assigned to each combination is a function of a number of parameters. The factors considered in assigning a score include the following: 1) the narrow lane residuals; 2) the refraction corrected residuals; 3) the L_1 and L_2 position separation (note that the matrix defined by the left hand side of equation (9), which is needed to compute the S matrix, can be used to compute the position updates or position separation simply by multiplying by the appropriately scaled search permutations); 4) the distance from the code solution; 5) the number of the wide-lane integers in the permutation which selected the larger (greater than 0.5) fractions; and 6) the number of narrow-lane integers in the permutation which selected the larger fractions.

The above factors are used to assign a score to each permutation which meets a number of criteria. Among the criteria is that the score be less than a threshold value, and

that threshold value is itself a function of the separation distance between the base station and the user. If no permutation is selected as the correct permutation, the scores are cumulated from epoch to epoch.

The best integer ambiguity permutation is assigned a differential score. This differential score is the difference between the second best permutation and the best. If there is no second best permutation which is acceptable then the differential score of the best permutation is the difference between the score threshold and the best permutation's score. If the differential score is large enough the best permutation is accepted as the correct integer ambiguity set. However, if there are fewer than seven satellites involved in the ambiguity search, the entire process is repeated and several repeated selections of the same integer ambiguity permutation is required for final acceptance as the correct set of integers.

While the above process is a bit complicated, when seven or more satellites are available, it often provides the correct integer ambiguity set in a single epoch. Identifying the wrong set of integers as the correct set typically occurs only a few times out of 1000 attempts. When at least seven satellites are available integer ambiguity failure is extremely rare.

TEST RESULTS

The RTK ambiguity search process described above has been incorporated into NavCom's dual-frequency OEM GPS receivers. However, in order to more easily evaluate the ambiguity resolution process, an offline version was also built. The offline capability allows one to perform scoring runs in which the ambiguity search process is repeated over and over. As soon as one set of integer ambiguity values is declared correct, the search process is reinitialized (which takes one epoch) and a new search is performed.

Figures 1 and 2 show the results of processing 10 hours of short baseline (approximately 10 meters) in a scoring run. 13,659 successful searches were conducted. 84% of the searches were accomplished in a single epoch.

Figure 3 and 4 show the results of processing approximately an hour and a half of data collected over a 5 kilometer baseline. 1,772 successful searches were conducted of which 82% were accomplished in a single epoch.

The navigation accuracy is shown in a bull's eye plot in Figure 5. The standard deviation in north, east and up were 4, 5 and 11 millimeters respectively.

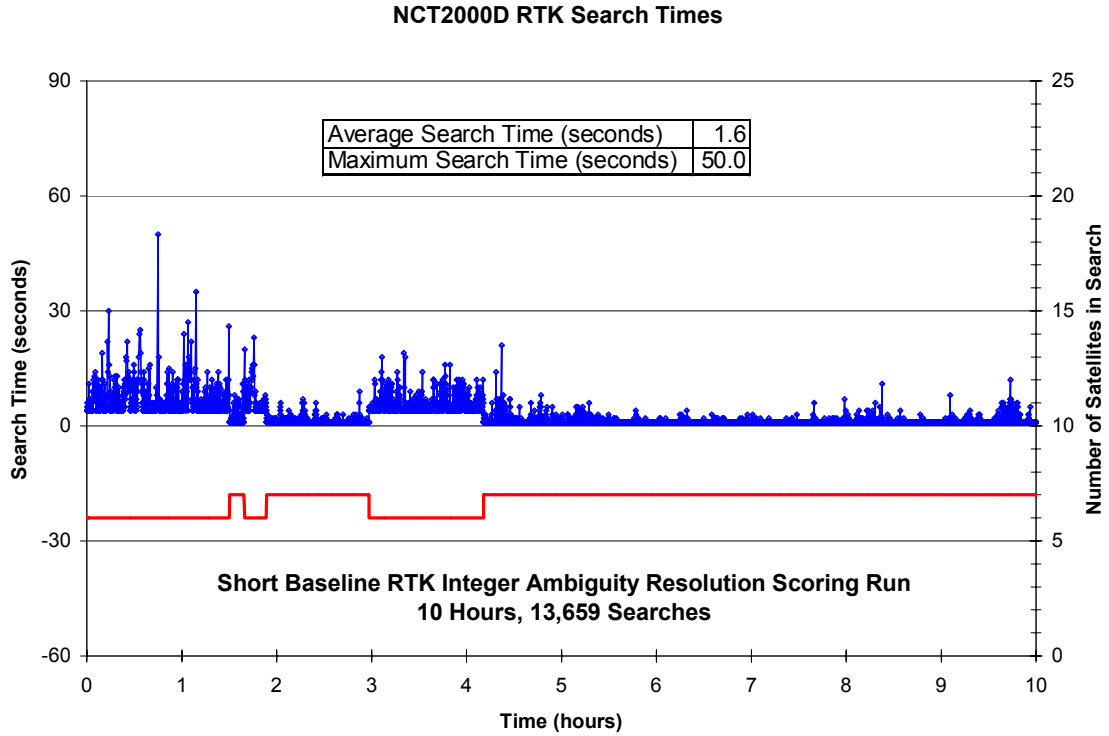


Figure 1. Short Baseline Scoring Run Search Time vs. Run Time

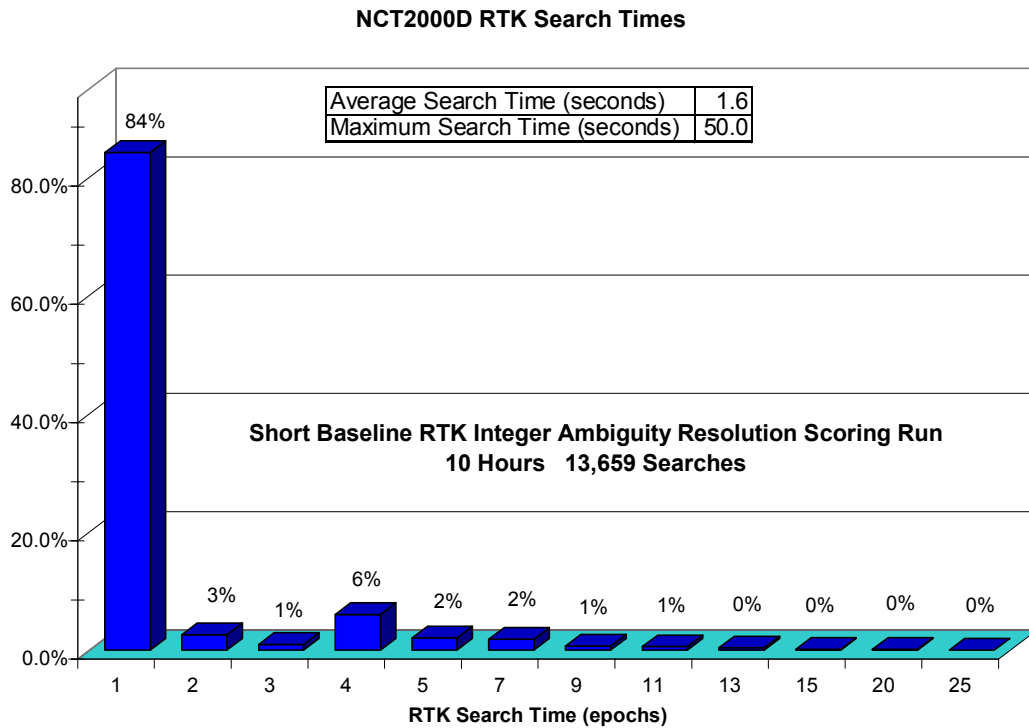


Figure 2. Short Baseline Scoring Run Histogram of Search Times

NCT2000D RTK Search Times

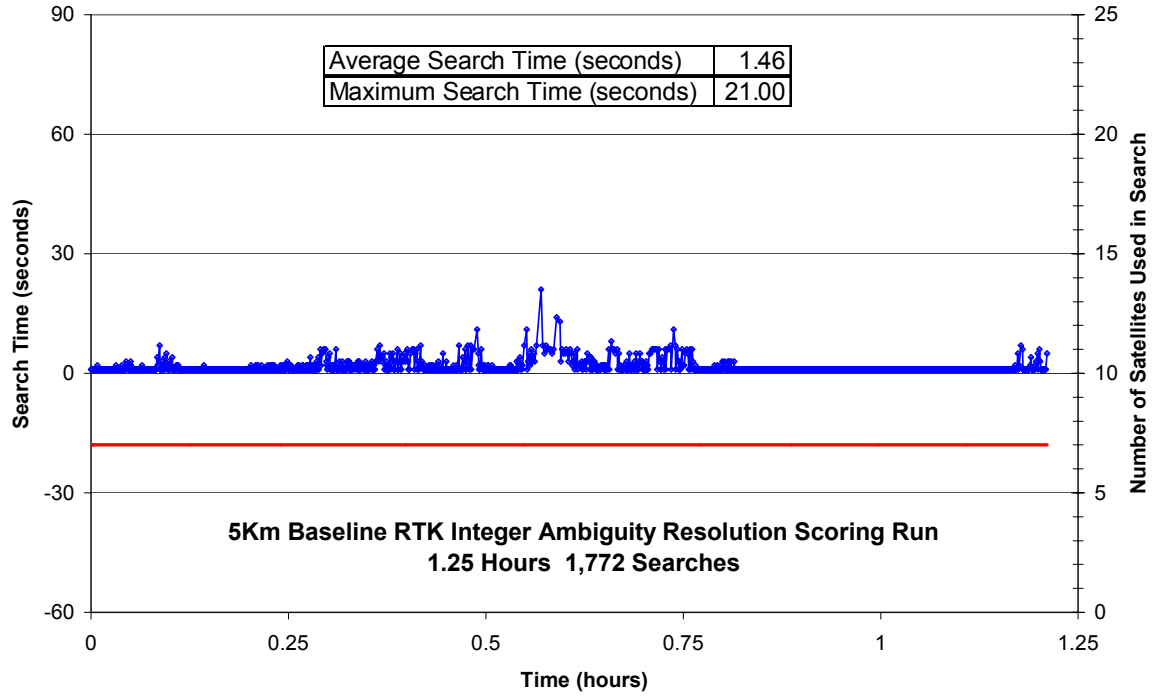


Figure 3. Five Kilometer Scoring Run

NCT2000D RTK Search Times

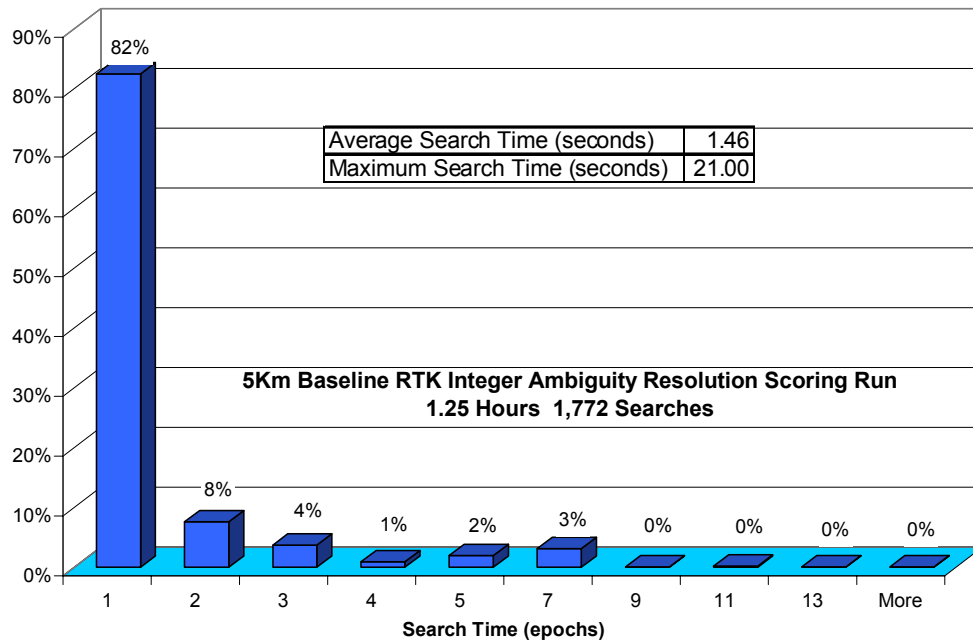


Figure 4. Five Kilometer Scoring Run

NavCom RTK Navigation 24 Hours on a Stationary Point

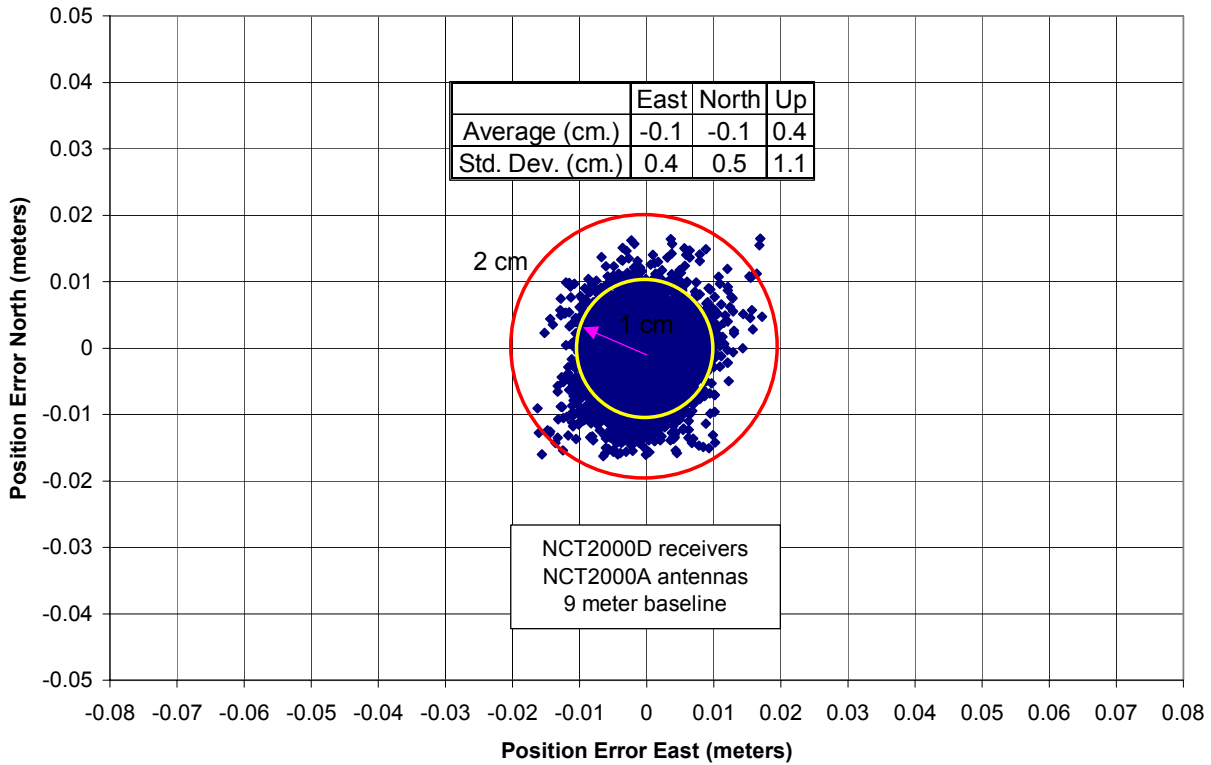


Figure 5. RTK Navigation Results – Horizontal Position Scatter Plot

CONCLUSION

A new approach to the ambiguity resolution process has been described. It is computationally efficient, in that a simple process is used to compute the residuals without first computing a position solution. Position solutions are computed (also using a simplified process) only when the residuals indicate the particular integer ambiguity permutation has a high probability of being the correct set. Using a wide-lane search before stepping to a narrow lane search, is also efficient in that fewer permutations are required to cover the same three-dimensional search region. The frequency-weighted code combination ensures a starting position with minimal multipath corruption and the longer the search takes the better the initial code solution becomes because of the code-smoothing employed. As shown by the test results, the search procedure is highly reliable and 80% or more of the searches are successfully completed in a single epoch.

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